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A BOLOMETRIC STUDY OF LIGHT STANDARDS.

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I.

Introduction.

THE two great essentials for a light standard are constancy during use and exact reproducibility.

It is evident even to a most casual observer that the standard candles which are in use do not possess the first of these essentials. The English standard candle has been charged with fluctuations of 40 per cent, and it is only by snuffing and measuring carefully the height of flame that anything approaching uniform results has been obtained.

The Carcel lamp has stood the test of time but little better than the best of standard candles. It has disadvantages entailed by the use of a light-absorbing chimney and a rapidly charring wick; while of its reproducibility, within a reasonable margin of error, there is serious question. Although the Carcel lamp furnishes a very steady source of light, it is known to show slow changes in the amount of light emitted, rising slowly to a maximum as the lamp and oil become heated, and then falling off with the charring of the wick.

Among the more modern standards that of Methven is faulty in that it requires a chimney, employs as an illuminant an indeter-

minate mixture of gases, and is affected also by variations in the pressure of the gas and by draughts of air in the room. The Vernon-Harcourt pentane burner and the Hefner-Alteneck amyl-acetate lamp, while avoiding this difficulty, run into others perhaps equally serious. The open flame of the latter is not only of a decidedly reddish hue, but is also extremely sensitive to draughts and vibrations of the air. On this account it is only with the greatest care that the height of the flame can be adjusted according to the prescription.

Many attempts have been made to determine the fluctuations of the various standard lights by comparing them by means of a photometer with a source of constant light, as, for example, a glow-lamp at constant voltage. This method is very faulty on account of fundamental difficulties of a physiological or psychological nature which are met in the use of even the most perfect photometer. It requires care and deliberation to make a photometer setting which can be relied upon as being correct within a narrow limit of error, while the fluctuations in the intensity of the light may be quite rapid.

If one seeks to determine these fluctuations by independent settings, succeeding each other as rapidly as reliable settings can be made, the extreme fluctuations may escape observation. The consequence is that no very definite idea can be obtained of the form of the curve which would be traced by the changes of the light intensity. On the other hand, if continuous observations are made with the photometer, it is possible to follow up the fluctuations only very tardily, irregularly, and imperfectly.

In any case the personal error of the observer enters so largely into photometric results that the perplexing question is continually arising, Have I really observed a fluctuation of the light, or have I merely made a bad setting? This remark applies, of course, only to the smaller fluctuations, and not to the large ones commonly observed in the case of candles.

In order to determine these fluctuations, an instrument is needed which will not only serve to measure radiant energy, but which will also respond instantly to any change in the amount of radiation meeting it. Such an instrument is found in the bolometer.

Suggestions as to the use of the bolometer in photometry have not been infrequent. Lummer and Kurlbaum[1] have described a large double-faced bolometer of platinum foil, constructed by them for photometric work. It is difficult to see how this bolometer, intended as it was to be used among the fickle draughts and changing temperatures of a photometer room, would serve to get a continuous curve of the variations of a light standard, by the method described below.

In order to get a complete knowledge of the performance of any light source, it is necessary to obtain a continuous record of its behavior during a considerable period of time. If we make the assumption that light radiation from a given source varies in direct ratio to the total radiation, a properly constructed bolometer furnishes a suitable instrument for obtaining the record desired. The bolometer strip must be so thin that its temperature will follow almost instantly all changes in the amount of radiation meeting it, and the galvanometer must be nearly dead beat. Then if the working conditions are such that it is certain that all observed galvanometer deflections are due to changes in the radiation from the light source, and to nothing else, a plat of galvanometer deflections will represent very truly the variations in the intensity of the light.

The proper adjustment of conditions involves the very careful exclusion of draughts of air from the bolometer strips and from the other parts of the apparatus, and the maintenance of a nearly constant temperature in the room. Moreover, it necessitates the taking of observations at such a time or place that the galvanometer shall not be subjected to any irregular magnetic influences. In order to fulfil these conditions, the apparatus described below was constructed.

II.

Description of Apparatus.

After numerous unsatisfactory experiments with gold leaf lightly plated with nickel, with pure nickel, and with the thinnest tin foil, the bolometer strips were made as follows: —

[1] Lummer und Kurlbaum, Weid. Ann., Vol. 46, p. 204; Zeitsch. für Instrumentenkunde, Vol. 12, p. 81.

A piece of Swedish iron wire, of No. 30 B. & S. gauge, was passed through jeweler's rollers until its thickness was about 0.045 mm., and its width 1.5 mm. It was then placed in dilute sulphuric acid, in which potassium bichromate had been dissolved, and a current was passed through it in such a manner as to dissolve the iron. The potassium bichromate was introduced into the solution to dispose of the hydrogen bubbles which would ordinarily have clung to the metallic surface and which would have caused it to be dissolved unevenly.

In this way a strip was obtained which was about 0.025 mm. in thickness and still moderately strong. From this strip were cut two pieces, each about 6 cm. in length, to constitute two arms of a Wheatstone's bridge.

To carry the strips so obtained, a light oblong frame, *F* (Fig. 1), of thin wood was made, and to it were fastened small bits of sheet brass, *b*, *b*, *b*, to which the strips and the copper wires intended to connect them with the other arms of the bridge could be soldered. The strips, *S*, *S'*, were then bent and placed over the frame, so that each strip crossed the frame twice. The free ends of each strip were displaced laterally from each other so that, when viewed from the front, the portion of the strip on one side of the frame hid only very little of the portion on the other side of the frame.

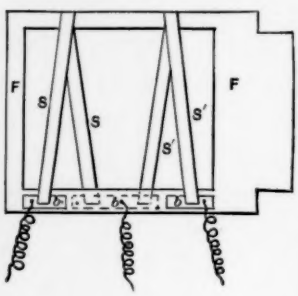


Fig. 1.

After the two strips had been arranged on the frame symmetrically with respect to each other, the one which was to receive radiation was carefully smoked on both sides. To accomplish this smoking without undue heating of the strip, a piece of sheet metal, through which a small hole had been punched, was held over a candle flame so that the flame was caused to smoke. The smoke passed through the hole, over which a tube was held to direct the current. The strip was passed back and forth over the top of the tube. In this way a very delicate strip can be blackened without injury. In their completed state the strips had a resistance of about 0.5 ohm each.

The frame holding the strips was mounted in a wooden box about 20 cm. long and 5 cm. × 6 cm. in cross-section. A number of suitable cardboard screens were placed in the front portion of the box to shield one of the strips from radiation and to protect both of them from draughts. To close the box at its other end a piece of bright tin was cut equal in size to the cross-section of the box. This was soldered fast to a heavy block of brass, to the other end of which was soldered another piece of tin which covered the end of the box. When this arrangement was placed in the box, the first piece of tin came close behind the bolometer strips, and its bright surface acted as a mirror in reflecting upon the back of the exposed strips many rays which would otherwise have been lost.

By this means the efficiency of the bolometer was nearly doubled. The tinned surface did not tarnish perceptibly during the course of the investigation. It was not smooth enough to reflect a distinct image, and the light reflected from it was to a large extent scattered. The use of a plane-surfaced mirror in such a position would not be allowable, since any slight change in the angle of incidence would cause a different amount of light to be reflected upon the bolometer strip. The use of the irregular-surfaced plate, since it diffuses the light, can scarcely affect the accuracy of the results to an appreciable degree in such work as has been done with this bolometer. Nevertheless, this arrangement is to be recommended only where great sensitiveness is desired rather than the most exact comparison of results.

Other sources of error which might affect the exact comparability of results have also been ignored. Among these may be mentioned the error which is introduced in assuming that the sensitiveness of the bolometer is independent of the absolute temperature of the strips. The brass block served to keep the temperature of the tin screen from rising to any marked degree above that of the air in the room, by conducting any excess of heat to the outer piece of tin, the surface of which was smoked.

The compensating resistance, forming the other arms of the bridge, was made of german-silver wire arranged inside a small wooden box. Fig. 2 gives the top and side views of this box.

The german-silver wire was strung back and forth on tacks nailed in the two sides of the box, as shown in the side view. The middle portion, *S*, of this wire was drawn tightly, lengthwise of the box, and a sliding contact piece, *C*, was arranged to press against this wire. This contact piece was carried on a little block of wood, *A*, running on ways, *F*, *F'*, and driven by a fine-threaded screw, *E*, which could be turned by means of a large disk head, *D*, outside of the box. One terminal of the battery (which consisted of two

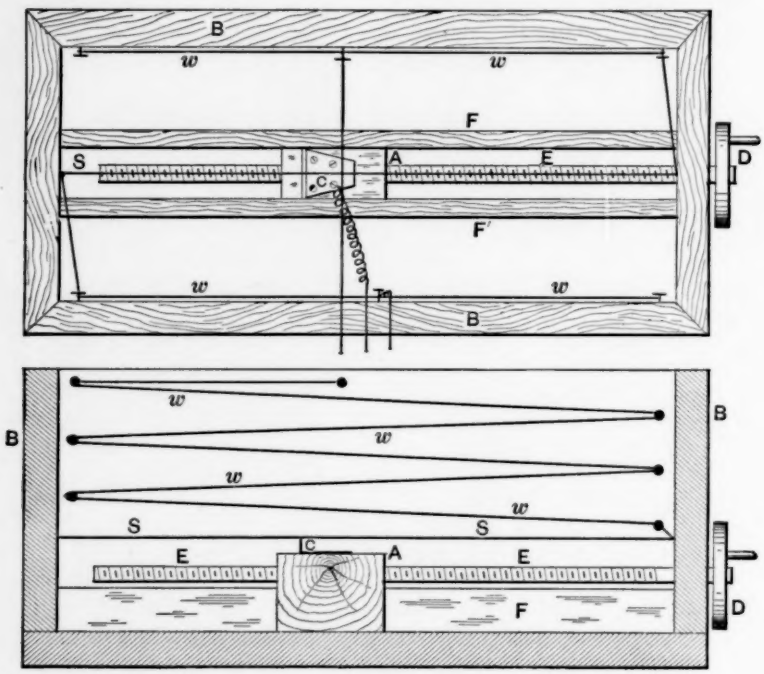


Fig. 2.

gravity cells in multiple with each other) was connected with corresponding ends of the bolometer strips. The other terminal was connected with the sliding contact piece just mentioned. In order to adjust the balance of the bridge, it was necessary only to turn the screw and so to move the contact piece along the stretched wire, until a position was found where no current flowed through the galvanometer. This method was found to be very delicate and convenient.

The compensating resistance was connected with the bolometer strips by means of copper wires. All metallic junctions which might give rise to thermal currents were, as far as possible, so arranged that currents produced in this way in any part of the bridge would be opposed by similar thermal currents produced in the opposite part of the bridge; this condition being secured by bringing corresponding thermo-junctions as near to each other as it was convenient to do.

The galvanometer employed was of the four-coil type. It was constructed by Professor W. S. Franklin, after the same general plan as has been followed by Snow,[1] Paschen,[2] and others. When the two front coils were in multiple with each other and the two rear coils similarly connected, and the two pairs were connected in series, the resistance was found to be 190 ohms. The moving parts consisted of four little magnets of piano wire, each about 5 mm. long, and a mirror of thin cover glass, 4 mm. wide by 7 mm. long, all mounted on a slender rod of glass and suspended by a very fine quartz fiber. Any oscillations of the needles were very strongly and effectively damped by the air resistance to the light-moving parts, — a very essential condition to the correct operation of the instrument when used to get the variations of a rapidly fluctuating source of radiant energy.

The scale was divided into 100 half-inch divisions, each of which was, in turn, divided into tenths. The distance of the scale from the galvanometer was 100 scale divisions, *i.e.* 50 inches. With the telescope used, fifths of the smallest divisions could be estimated. In speaking of scale divisions, the half-inch divisions will always be meant.

To determine at any time the sensitiveness of the arrangement, a plan was followed which was first proposed by Knut Ångström.[3] In multiple with the exposed bolometer strip was placed a resistance of 1000 ohms. By removing a plug and throwing 5000 ohms more into the shunt circuit, the galvanometer needle was deflected in the same direction as when the strip was heated. Only the

[1] B. W. Snow, Wied. Ann., Vol. 47, p. 213; Phys. Review, Vol. 1, p. 2.

[2] F. Paschen, Wied. Ann., Vol. 48, p. 272; Zeitschr. für Instrumentenkunde, Vol. 13, 1893, p. 13.

[3] Ångström, Oefvers. af kongl. Vetenskaps-Akad. Förhandl., 1888, Vol. 6, p. 379.

first throws of the needle were taken in observations for sensitiveness, — a procedure which a comparison of first throws and final deflections, at different degrees of sensitiveness, showed to be justifiable. The ratio of first throws to permanent deflections was found to be 1.5.

No attempt has been made to correct the determinations of sensitiveness for any variation of the ratio of resistance of the bolometer strip and that of the shunt, due to temperature changes. This would be a simple correction to apply if only one could ascertain the temperature of the strips at any time. Since, however, this temperature is a function not only of the temperature of the room, but also of the strength of the main current and of the radiating power of the surface of the strip, a direct determination of it is impracticable. A value for this correction combined with the correction for the change in sensitiveness due to change in absolute temperature of the strip, could have been ascertained by comparing deflections given by a source of constant radiation, under different conditions of room temperature and current strength; but for the present work such refinements were deemed unnecessary, since the error introduced by not applying these corrections would form only a small proportional part of the quantities to be measured.

For observations requiring only a low degree of sensitiveness, the galvanometer coils were connected in the multiple-series arrangement described above. For higher degrees of sensitiveness all the coils were put in multiple, the increase in sensitiveness produced by this change being about twofold. With either connection the fine adjustment of sensitiveness was obtained by changing the distance of a controlling magnet. In nearly all cases the sensitiveness of the bolometer was such as to permit the controlling magnet to be placed so as to strengthen the earth's field, and the period of the galvanometer needle was correspondingly short.

The constants of the galvanometer and of the bolometer were as follows: With the galvanometer coils in the multiple-series connection and the sensitiveness adjusted to give a throw of the needle of 16.2 scale divisions on putting the 5000 ohms in the shunt (this being the standard sensitiveness for candles), the period

of the needles was 6 seconds for a complete vibration. A deflection of one scale division corresponded to a current of 68×10^{-10} amperes.

This deflection of one scale-division corresponded to a temperature rise in the bolometer strips of 0°.00657 C. If we reduce this to millimeter divisions on a scale placed at a distance of 1 m. from the mirror, we see that one millimeter deflection corresponded to a current of 68×10^{-11} amperes, and the corresponding rise in temperature of the strip was 0°.00066 C.

This temperature sensitiveness is much smaller than has usually been employed in bolometer work, but it was amply sufficient for the purpose. That the bolometer itself was one of high sensitiveness is evident from the fact that this degree of sensitiveness was attained with galvanometer needles swinging in a strengthened field and with a galvanometer of 190 ohms resistance. The conditions for maximum sensitiveness of the bridge would have required a galvanometer resistance of only 0.5 ohms.

The reason for the great sensitiveness lay in the nature of the strips employed. Their area was considerable, the temperature coefficient of the iron was high, and the current through it was large, ranging from 0.15 to 0.20 amperes, the size of the strips permitting the use of a large current without undue heating. As a result of the strong field in which the galvanometer needles swung, the drift due to magnetic changes was usually imperceptible.

The bolometer, compensating resistance, and battery were all placed in an interior room, with thick brick walls, and having communication with the outer room only by a door, *D*, Fig. 3.

The temperature of this room changed very slowly, and it was quite free from draughts. The bolometer box, *B*, was placed upon a shelf fastened to the door of the room, and looked out, through a hole in the door, upon the outer room. A double screen, *S*, of tin, arranged to slide up and down on the outside of this door, covered up the bolometer strips when desired. The box, *B'*, containing the compensating resistance, was also fastened to the inside of the door. The end of the box through which the screw projected, fitted into a hole in the door, so that the screw

could be turned and the bridge balanced from the outer room. The wires, *g*,*g*, led to the galvanometer, and *s*, *s* led to the box which was in multiple with the exposed strip. For further protection from draughts the bolometer box and connecting wires were covered with cotton waste.

Under this arrangement it was necessary to enter the inner room only occasionally to attend to the batteries, and, therefore, the bolometric resistances were subjected to no sudden changes

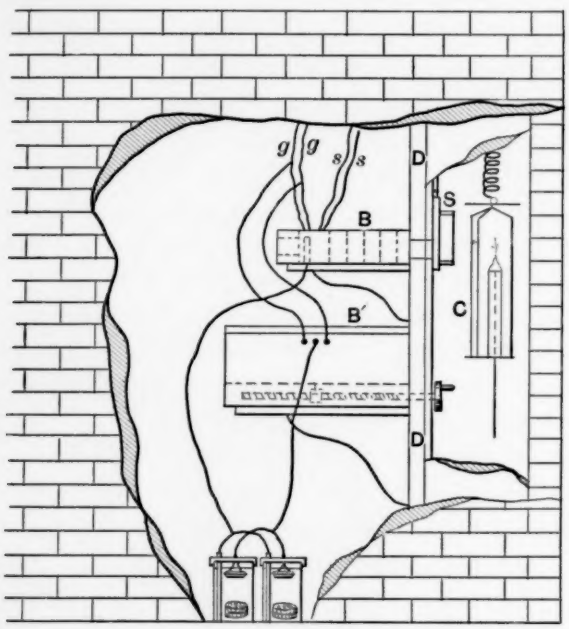


Fig. 3.

of temperature. As a consequence, drift due to changes of temperature of these resistances was very small and quite regular. Unfortunately, however, it was necessary to place the box containing the resistance which was in multiple with the exposed strip in the outer room, near the galvanometer telescope. The resistance of this box, and of the wires leading to it, changed slightly with temperature changes in the outer room. As a consequence a slight and regular drift would always ensue upon lighting the gas in the outer room preparatory to beginning work.

Since only a very small proportion of the current flowed through this outer circuit, this drift was never large, and would gradually cease as the temperature of the room became constant. Various expedients were adopted for minimizing this temperature change; such as opening a window slightly to admit enough cold air to counterbalance the heating due to the gaslights; closing the valves in the radiator pipes, etc. In every case correction was made for the drift and for any change of sensitiveness which might take place, on the assumption that the change had been a perfectly regular one. Whenever it would seem that this assumption could result in any considerable error, the observations were rejected.

In getting the variations of candles, a special device was required to keep the top of the burning candle at constant height in front of the bolometer strip. A suggestion of Mr. C. H. Bierbaum, M.E., resulted in the construction of the following simple and effective arrangement. A spiral spring, about 60 cm. long and 4 cm. in diameter, was attached to a small scale-pan, and the spring was cut off to such a length that when a candle was put on the scale-pan, the elongation was just equal to the length of the candle. The spring would then take up as fast as the top of the candle was lowered by burning. A small piece of sheet metal served to protect the spring from the heat of the candle. In order to keep the scale-pan from swinging sidewise and from oscillating up and down, a couple of wires were passed vertically through holes on opposite sides of it, and served as loosely fitting guides, which, without interfering with the take-up of the spring, effectually damped any vibrations. The adjustment of a spring to any candle could be made in a few minutes with sufficient accuracy so that the height of the top of a candle burning on the scale-pan would not vary over 1 mm. in an hour. *C*, Fig. 3, shows this arrangement in place.

III.

Method of Taking Observations.

All determinations of the variations of standards were made at times when the laboratory and its surroundings were very quiet. Most of them were made between the hours of seven and twelve in the evening; a few were made on holidays, when the laboratory was closed for general work. Before beginning a set of observations the sensitiveness was adjusted, and the galvanometer was carefully watched for a considerable time to get the amount of its swings due to currents of air about the bolometer strips or to changes in the earth's field. The sensitiveness was also tested from time to time in order to find whether this was variable or not. If swings of the galvanometer greater in amount than two-tenths of a scale division, or any irregular changes in sensitiveness were detected, no run was attempted. Since two-tenths of a scale division amounted to only about 0.5 per cent of the total deflection in most cases, and since the variation in the light sources themselves were many times two-tenths of a scale division in the case of all the standards except the Carcel lamp, this amount of error was considered entirely negligible.

After taking these preliminary observations, which usually required about an hour's time, the bolometer was exposed to the source of light. After waiting a few minutes for the strip and the surrounding parts to be thoroughly heated, the galvanometer deflections were read rapidly by one person, and were plotted by another as fast as they were read, the time being taken from a watch. In this way curves were traced which represent very truly all the changes in the radiation of the light source. This method of recording observations was found to be much preferable to the ordinary one of noting galvanometer readings and plotting them afterwards, both in point of saving of time and in accuracy attainable. It made possible, not only more numerous readings, but also the giving of oral instructions to the recorder, and the consequent plotting of curves which, in correctness of their minor as well as of their major details, would be but little surpassed by a photographic record.

An unfortunate feature in these curves is that the scale of abscissas, representing times, is so small. As a result of this it has been difficult to represent with the greatest accuracy the true slope of those portions of the curves which correspond to very rapid changes in radiation. In examining the curve it should be borne in mind that a very steep line may cover several seconds of time, and that it is quite possible that a photometric setting might be made during the time that a candle is executing just such a variation as is represented by the steepest parts of one of the candle curves. The number of galvanometer readings plotted during the space of five minutes was usually from 50 to 100.

At the end of a run the strips and surrounding parts were given time completely to lose the heat imparted to them, after which readings for zero and sensitiveness were taken. To correct the curves for changes in zero and in sensitiveness, a plan was followed which was justified only by the fact that the error introduced by employing it was entirely insignificant. Change in sensitiveness was reduced to ordinates of the curve by means of the equation: —

$$\frac{\text{change in sensitiveness}}{\text{initial sensitiveness}} \times \text{approximate mean ordinate} = \text{correction to ordinate.}$$

This was combined with the drift, and a point was placed on the cross-section paper having its ordinate such as to represent the combined effect of drift and change of sensitiveness, its abscissa being the time of taking observations. This point was joined by a straight line with a point on the axis of X at the time of taking the initial zero readings, thus assuming that the drift and change of sensitiveness had been regular during the run. Then, if the initial sensitiveness happened to be unequal to the sensitiveness which was taken as standard, by an equation similar to the one given above, a correction to all the ordinates was computed to reduce the curve to the standard sensitiveness. If the distance of the light source from the strip was other than that taken as standard, a similar correction was computed for this. These two reductions were then combined, and a line was drawn parallel to the first correction line and distant from it by an amount equal to

these combined corrections. The mean ordinates of the curves, reckoned from the new base line so constituted, represent the relative quantities of radiation of the sources of light. The curves were then traced on bond paper, the axis of abscissas on which was put so as to fall on the correction line on the original cross-section paper. Since the corrections which have been spoken of above were all small (except in the case of curves XVI. and XVI. (*a*)), neither the proportionality of the variations to the total deflection, nor the slope of the lines of the curves, has been materially affected.

Before setting out on the work it was feared that the inertia of the moving parts of the galvanometer might have a decided effect on the accuracy of the results obtained, in that it would tend to exaggerate the rapid changes. This fear was found to be groundless. In the first place, the galvanometer was, as has been said, very nearly dead beat. Although the bolometer strip followed almost instantly every fluctuation of the source of radiation, the fluctuations themselves were not usually so rapid that the galvanometer needles were driven with any considerable velocity. Consequently the tendency of the needles to oscillate in their natural period was very slight. That the inertia of the moving parts had very little, if any, influence on the shape of the curves was shown by the fact that at the end of most swings the needle would pause for some seconds before beginning its return; while it rarely happened that the angular velocity did not change during a swing.

It has been impossible to draw the curves so as to show clearly these variations which involved only a few seconds of time, since, as has been said, the scale to which the time abscissas are plotted is very small compared with the scale of ordinates. These important peculiarities of the movements of the galvanometer needle were clearly perceptible, however, to one who observed the galvanometer deflections for only a little time; while their character made it perfectly evident that the swings of the galvanometer needles in their natural period exercised only an inappreciable influence on the slopes of the curves and the magnitudes of the variations recorded.

On the plates the smallest divisions of abscissas represent five minutes of time. The corresponding divisions of ordinates are five divisions of the galvanometer scale. In order to economize space, no attention has been given to the height of the curves relative to the base line on the plates. The total ordinates for each curve are indicated by such a symbol as this $\left.\begin{matrix}\text{I}\\\text{II}\end{matrix}\right\}40$, indicating that the true X-axis for curves I. and II. should be placed 10 divisions below the base line on the plate. Consequently on the various plates curves having the same ordinates are plotted one above the other, the true ordinate being indicated in the way mentioned above.

The data by which the curves have been corrected are given in Table I.

Table II. gives, in its first column, the approximate abscissas of the portion of the curve in question, and in its second column the mean ordinates of these portions. The third column contains the deviations of the mean ordinates of the portions from the mean ordinate of the whole curve. The fourth column contains these deviations reduced to percentages of the whole. The fifth contains similar deviations from the mean ordinate of all the curves of the particular standard under consideration, and in the sixth these also are reduced to percentages.

The standard distance and sensitiveness for the candle and Hefner lamp were 26 cm. and 16.2, respectively; for the Carcel lamp they were 56.7 cm. and 15, and for the Methven screen they were 50 cm. and 40, respectively.

TABLE I.

	Number of curve.	Times of taking zero and sensitiveness readings.	Time of lighting.	Time of taking the curve.	Hourly rate of consumption.	Distance from bolometer.	Initial sensitiveness.	Correction for		Reduction to standard		
								Drift.	Change of sensitiveness.	Sensitiveness.	Distance.	Rate.
		h. m. h. m.	h. m.	h. m. h. m.	grams.	cm.						
English Candle.	I.	8 47 and 10 01	8 50	8 50 to 9 35	7.910	25.3	16.2	+0.9	+0.2	0	−2.3	−0.70
	II.	9 41 10 50	9 33	9 50 10 35	7.767	26.0	16.1	−0.4	0	+0.3	0	+0.05
	III.	8 00 9 15	8 00	8 00 9 00	7.842	26.0	16.1	−2.9	+0.8	+0.3	0	−0.34
	III. (*a*)	9 38 10 30	8 00	9 45 10 15	7.842	26.0	16.08	+0.3	0	+0.3	0	−0.34
	IV.	7 50 9 05	7 50	7 50 8 50	7.692	25.3	16.15	+1.0	−0.1	+0.1	−2.1	+0.45
	IV. (*a*)	9 35 10 30	7 50	9 45 10 15	7.692	25.3	16.4	−0.1	0	−0.5	−2.3	+0.45
German Candle.	V.	7 55 9 10	7 55	7 55 8 55	8.60	25.3	16.27	+1.1	−1.6	−0.3	−2.8	—
	V. (*a*)	9 50 10 40	7 55	9 55 10 25	8.60	25.3	16.3	−1.7	+1.0	−0.3	−2.9	—
	V. (*b*)	8 21 9 25	8 10	8 25 9 10	—	26.0	16.2	−5.7	+4.7	0	0	—
	VI.	8 05 9 20	8 05	8 05 9 05	—	26.0	16.15	+1.3	−0.3	+0.15	0	—
	VI. (*a*)	9 30 10 31	8 05	9 46 10 16	—	26.0	16.2	−0.3	−0.15	0	0	—

Hefner Lamp.	VII.	8 05	8 57	7 15	8 10	8 40	—	25.0	16.33	−3.1	+1.8	−0.4	−2.2	—
	VII. (*a*)	9 05	10 00	7 15	9 10	9 45	—	25.0	16.1	−1.3	−1.9	+0.3	−2.3	—
	VIII.	10 17	11 00	9 57	10 25	10 45	—	26.0	16.2	−3.2	+0.5	0	0	—
	IX.	8 08	9 05	7 13	8 20	8 50	—	26.0	16.2	−0.3	+0.1	0	0	—
	X.	8 42	9 20	—	8 50	9 00	—	36.8	32.5	−0.3	−0.3	−0.1	0	—
	XI.	9 08	10 26	8 55	9 14	9 34	—	26.0	16.2	−3.0	+0.8	0	0	—
	XI. (*a*)	9 08	10 26	8 55	9 38	9 40	—	26.0	16.2	−3.0	+0.8	0	0	—
	XI. (*b*)	9 08	10 26	8 55	9 46½	9 49½	—	26.0	16.2	−3.0	+0.8	0	0	—
	XI. (*c*)	9 08	10 26	8 55	9 55	9 58	—	26.0	16.2	−3.0	+0.8	0	0	—
	XII.	7 53	8 35	7 40	8 00	8 20	—	26.0	16.2	−2.3	+0.4	0	0	—
	XII. (*a*)	8 40	9 25	7 40	8 50	9 10	—	26.0	16.2	−0.7	0	0	0	—
	XIII.	10 11	10 50	9 38	10 15	10 35	—	26.0	16.2	−0.5	+1.3	0	0	—
	XIV.	10 00	10 35	9 53	10 08	10 18	—	26.0	16.2	+0.7	0	0	0	—
Carcel Lamp.	XV.	10 14	11 03	9 40	10 18	10 48	47.05	56.7	15.0	+0.7	−0.4	0	0	−4.5
	XVI.	8 38	9 38	8 10	8 45	9 25	43.05	56.7	28.2	+2.8	−0.4	−33.0	0	−0.7
	XVI. (*a*)	10 00	10 38	8 10	10 03	10 23	44.14	56.7	28.4	0	+0.2	−35.6	0	−1.5
	XVII.	8 13	8 55	7 59	8 20	8 40	46.85	56.7	15.03	+0.6	0	−0.1	0	−3.0
	XVII. (*a*)	10 54	11 20	7 59	10 59	11 09	40.00	56.7	15.0	0	0	0	0	+1.3
Methven Screen.	XVIII.	9 15	10 06	—	9 21	9 51	—	49.7	39.8	+4.7	−0.2	+0.1	−0.3	—
	XIX.	9 22	10 00	—	9 30	9 50	—	50.0	39.6	+2.5	−1.0	+0.2	0	—
	XX.	9 00	9 51	—	9 06	9 36	—	50.0	40.2	−0.5	+0.2	−0.2	0	—
	XXI.	8 17	8 59	—	8 24	8 44	—	50.0	40.1	+0.6	+0.1	−0.1	0	—
	XXII.	9 01	9 37	—	9 09	9 19	—	50.0	40.0	+0.6	0	0	0	—

TABLE II.

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the English candle curves.	Deviations from general mean reduced to percentages.
CURVE I.	15–20	41.68	−1.04	−2.43	+0.59	+1.44
	20–25	44.68	+1.96	+4.58	+3.59	+8.72
	25–30	40.84	−1.88	−4.40	−0.25	−0.61
	30–35	43.39	+0.67	+1.57	+2.30	+5.59
	35–40	43.71	+0.99	+2.32	+2.62	+6.37
	40–45	42.04	−0.68	−1.59	+0.95	+2.31
		42.72	Correction for rate of burning = − 0.70.			
CURVE II.	45–50	39.24	−1.77	−4.32	−1.85	−4.50
	50–55	37.85	−3.16	−7.70	−3.24	−7.87
	55–60	39.12	−1.89	−4.61	−1.97	−4.79
	60–65	40.44	−0.57	−1.39	−0.65	−1.58
	65–70	40.76	−0.25	−0.61	−0.33	−0.80
	70–75	42.85	+1.84	+4.48	+1.76	+4.28
	75–80	43.52	+2.51	+6.12	+2.43	+5.92
	80–85	43.20	+2.19	+5.34	+2.11	+5.13
	85–90	42.11	+1.10	+2.68	+1.02	+2.48
		41.01	Correction for rate of burning = +0.05.			
CURVE III.	15–20	43.04	−0.54	−1.25	+1.95	+4.74
	20–25	42.88	−0.70	−1.62	+1.79	+4.35
	25–30	42.82	−0.76	−1.76	+1.73	+4.21
	30–35	42.18	−1.40	−3.23	+1.09	+2.65
	35–40	43.23	−0.35	−0.80	+2.14	+5.20
	40–45	44.80	+1.22	+2.80	+3.71	+9.02
	45–50	43.58	0	0	+2.49	+6.06
	50–55	44.30	+0.72	+1.65	+3.21	+7.80
	55–60	45.38	+1.80	+4.13	+4.29	+10.45
		43.58	Correction for rate of burning = −0.34.			
CURVE III. (*a*).	60–65	43.25	+0.23	+0.53	+2.16	+5.25
	65–70	43.52	+0.50	+1.16	+2.43	+5.91
	70–75	42.82	−0.20	−0.47	+1.73	+4.21
	75–80	43.68	+0.66	+1.54	+2.59	+6.30
	80–85	42.82	−0.20	−0.47	+1.73	+4.21
	85–90	42.04	−0.98	−2.28	+0.95	+2.31
		43.02	Correction for rate of burning = −0.34.			

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the English candle curves.	Deviations from general mean reduced to percentages.
CURVE IV.	15–20	38.65	+1.42	+3.82	−2.44	−5.94
	20–25	39.84	+2.61	+7.01	−1.25	−3.04
	25–30	38.07	+0.84	+2.26	−3.02	−7.35
	30–35	35.76	−1.47	−3.87	−5.33	−12.95
	35–40	35.72	−1.51	−4.05	−5.37	−13.08
	40–45	37.04	−0.19	−0.51	−4.05	−9.84
	45–50	35.84	−1.39	−3.75	−5.25	−12.78
	50–55	37.16	−0.07	−0.02	−3.93	−9.55
	55–60	36.96	−0.27	−0.72	−4.13	−10.05
		37.23	Correction for rate of burning = +0.45.			
CURVE IV. (*a*).	60–65	38.26	−1.47	−3.70	−2.83	−6.88
	65–70	39.04	−0.69	−1.74	−2.05	−4.99
	70–75	38.26	−1.47	−3.70	−2.83	−6.88
	75–80	40.84	+1.11	+2.80	−0.25	−0.62
	80–85	40.84	+1.11	+2.80	−0.25	−0.62
	85–90	41.16	+1.43	+3.60	+0.07	+0.17
		39.73	Correction for rate of burning = +0.45.			

Mean ordinate of all the English candle curves is 41.09.

Mean ordinate of all the English candle curves corrected for rate of burning is 41.05.

Mean ordinate of all the English candle curves corrected for rate and reduced to true deflections is 41.06.

					German candle curves.	
CURVE V.	15–20	49.48	−0.18	−0.36	−1.03	−2.05
	20–25	48.62	−1.04	−2.10	−1.89	−3.77
	25–30	49.20	−0.46	−0.92	−1.31	−2.60
	30–35	49.44	−0.22	−0.44	−1.07	−2.12
	35–40	49.40	−0.26	−0.52	−1.11	−2.20
	40–45	49.20	−0.46	−0.92	−1.31	−2.60
	45–50	49.52	−0.14	−0.28	−0.99	−1.96
	50–55	51.96	+2.30	+4.64	+1.45	+2.88
	55–60	50.12	+0.46	+0.92	−0.39	−0.77
		49.66				

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the German candle curves.	Deviations from general mean reduced to percentages.
CURVE V. (*a*).	60–65	52.80	+1.43	+2.78	+2.29	+4.53
	65–70	51.00	−0.37	−0.72	+0.49	+0.97
	70–75	49.44	−1.93	−3.76	−1.07	−2.12
	75–80	50.24	−1.13	−2.20	−0.27	−0.53
	80–85	50.68	−0.69	−1.35	+0.17	+0.34
	85–90	54.08	+2.71	+5.27	+3.57	+7.07
		51.37				
CURVE V. (*b*).	50–55	53.14	−0.78	−1.45	+2.63	+5.21
	55–60	55.41	+1.49	+2.77	+4.90	+9.71
	60–65	52.53	−1.39	−2.58	+2.02	+4.00
	65–70	52.82	−1.10	−1.88	+2.31	+4.58
	70–75	52.62	−1.30	−2.40	+2.11	+4.18
	75–80	53.30	−0.62	−1.15	+2.79	+5.53
	80–85	55.22	+1.30	+2.42	+4.71	+9.32
	85–90	56.33	+2.41	+4.47	+5.82	+11.52
		53.92				
CURVE VI.	15–20	45.24	−3.67	−7.35	−5.27	−10.43
	20–25	46.63	−2.28	−4.62	−3.88	−7.69
	25–30	48.17	−0.74	−1.51	−2.34	−4.63
	30–35	49.20	+0.29	+0.59	−1.31	−2.60
	35–40	49.84	+0.93	+1.90	−0.67	−1.33
	40–45	49.52	+0.61	+1.25	−0.99	−1.96
	45–50	49.88	+0.97	+1.99	−0.63	−1.25
	50–55	50.20	+1.29	+2.64	−0.31	−0.61
	55–60	57.48	+2.57	+5.16	+0.97	+1.92
		48.91				
CURVE VI. (*a*).	60–65	48.93	+0.14	+0.28	−1.58	−3.13
	65–70	49.20	+0.41	+0.84	−1.31	−2.60
	70–75	47.78	−1.01	−2.07	−2.73	−5.40
	75–80	48.52	−0.27	−0.55	−1.99	−3.94
	80–85	48.97	+0.18	+0.37	−1.54	−3.05
	85–90	49.36	+0.57	+1.17	−1.15	−2.28
		48.79				

Mean ordinate of all German candle curves is 50.51
Reduced to true deflections = 50.40

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Hefner lamp curves.	Deviations from general mean reduced to percentages.
CURVE VII.	0–5	38.81	−0.50	−1.27	+0.17	+0.44
	5–10	40.02	+0.71	+1.81	+1.38	+3.57
	10–15	39.64	+0.33	+0.84	+1.00	+2.59
	15–20	39.19	−0.12	−0.31	+0.55	+1.42
	20–25	39.03	−0.28	−0.71	+0.39	+1.01
	25–30	39.19	−0.12	−0.31	+0.55	+1.42
		39.31				
CURVE VII. (*a*).	30–35	39.96	+0.44	+1.11	+1.32	+3.42
	35–40	40.27	+0.75	+1.90	+1.63	+4.22
	40–45	39.83	+0.31	+0.79	+1.19	+3.08
	45–50	39.54	+0.02	+0.05	+0.90	+2.33
	50–55	39.09	−0.43	−1.09	+0.45	+1.16
	55–60	39.03	−0.49	−1.24	+0.39	+1.01
	60–65	39.19	−0.33	−0.84	+0.55	+1.42
	65–70	39.27	−0.25	−0.63	+0.63	+1.63
		39.52				
CURVE VIII.	70–75	38.91	−1.12	−2.80	+0.27	+0.70
	75–80	39.83	−0.20	−0.50	+1.19	+3.08
	80–85	40.54	+0.51	+1.28	+1.90	+4.92
	85–90	40.86	+0.83	+2.08	+2.22	+5.74
		40.03				
CURVE IX.	0–5	38.26	−0.54	−1.39	−0.38	−0.98
	5–10	38.62	−0.18	−0.46	−0.02	−0.05
	10–15	38.97	+0.17	+0.44	+0.33	+0.85
	15–20	38.87	+0.07	+0.18	+0.23	+0.59
	20–25	39.04	+0.24	+0.62	+0.40	+1.03
	25–30	39.03	+0.23	+0.59	+0.39	+1.01
		38.80				
CURVE X.	30–35	38.89	0	0	+0.25	+0.65
	35–40	39.16	+0.27	+0.70	+0.52	+1.34
	40–45	38.62	−0.27	−0.70	−0.02	−0.05
		38.89				

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Hefner lamp curves.	Deviations from general mean reduced to percentages.
CURVE XI.	45–50	39.42	−0.83	−2.06	+0.78	+2.02
	50–55	40.50	−0.10	−0.25	+1.51	+3.91
	55–60	40.50	+0.25	+0.62	+1.86	+4.81
	60–65	40.92	+0.67	+1.67	+2.28	+5.90
		40.25				
CURVES XI. (*a*), (*b*), and (*c*).	70	39.37	−0.24	−0.61	+0.73	+1.89
	75–80	40.00	+0.39	+0.98	+1.36	+3.52
	85–90	39.46	−0.15	−0.38	+0.82	+2.12
		39.61				
CURVE XII.	0–5	36.47	−0.90	−2.27	−2.17	−5.62
	5–10	37.21	−0.16	−0.43	−1.43	−3.70
	10–15	37.85	+0.48	+1.28	−0.79	−2.05
	15–20	37.94	+0.57	+1.52	−0.70	−1.81
		37.37				
CURVE XII. (*a*).	25–30	35.53	−0.67	−1.85	−3.11	−8.03
	30–35	35.73	−0.47	−1.30	−2.91	−7.52
	35–40	36.38	+0.18	+0.50	−2.26	−5.84
	40–45	37.18	+0.98	+2.71	−1.46	−3.78
		36.20				
CURVE XIII.	50–55	38.84	+0.15	+0.39	+0.20	+0.52
	55–60	38.74	+0.05	+0.13	+0.10	+0.26
	60–65	38.20	−0.49	−1.27	−0.44	−1.14
	65–70	39.00	+0.31	+0.80	+0.36	+0.93
		38.69				
CURVE XIV.	75–80	38.46	−0.16	−0.41	−0.18	−0.47
	80–85	38.78	+0.16	+0.41	+0.14	+0.36
		38.62				

Mean ordinate of first five minutes of Hefner lamp curves = 38.645.
Reduced to true deflections = 38.66.

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Carcel lamp curves.	Deviations from general mean reduced to percentages.
CURVE XV.	0-5	27.11	−0.49	−1.78	−1.16	−4.11
	5-10	27.37	−0.23	−0.83	−0.90	−3.19
	10-15	27.62	+0.02	+0.07	−0.65	−2.30
	15-20	27.68	+0.08	+0.29	−0.59	−2.09
	20-25	27.85	+0.25	+0.91	−0.42	−1.49
	25-30	27.98	+0.38	+1.34	−0.29	−1.03
		27.60				
CURVE XVI.	0-5	29.00	−0.37	−1.26	+0.73	+2.58
	5-10	29.09	−0.28	−0.95	+0.82	+2.90
	10-15	29.19	−0.18	−0.61	+0.92	+3.26
	15-20	29.32	−0.05	−0.17	+1.05	+3.72
	20-25	29.35	−0.02	−0.07	+1.08	+3.83
	25-30	29.61	+0.24	+0.82	+1.34	+4.74
	30-35	29.61	+0.24	+0.82	+1.34	+4.74
	35-40	29.76	+0.39	+1.33	+1.49	+5.28
		29.37				
CURVE XVI. (*a*).	40-45	29.00	+0.37	+1.29	+0.73	+2.59
	45-50	28.75	+0.12	+0.42	+0.48	+1.70
	50-55	28.65	+0.02	+0.07	+0.38	+1.34
	55-60	28.14	−0.49	−1.72	−0.13	−0.46
		28.63				
CURVE XVII.	60-65	25.90	−0.92	−3.44	−2.37	−8.39
	65-70	26.86	+0.04	+0.15	−1.41	−4.97
	70-75	27.27	+0.45	+1.68	−1.00	−3.54
	75-80	27.27	+0.45	+1.68	−1.00	−3.54
		26.82				
CURVE XVII. (*a*).	80-85	27.82	−0.24	−0.86	−0.45	−1.59
	85-90	28.30	+0.24	+0.86	+0.03	+0.11
		28.06				

TABLE II. (*continued*).

	Time: Minutes on curve.	Mean ordinates and their mean.	Deviations from the mean ordinate of the curve.	Deviations reduced to percentages.	Deviations from mean ordinate of all the Carcel lamp curves.	Deviations from general mean reduced to percentages.
CURVE XVIII.	0–5	25.95	+0.09	+0.35	−0.66	−2.48
	5–10	25.82	−0.04	−0.15	−0.79	−2.97
	10–15	25.57	−0.29	−1.12	−1.04	−3.91
	15–20	25.57	−0.29	−1.12	−1.04	−3.91
	20–25	25.95	+0.09	+0.35	−0.66	−2.48
	25–30	26.31	+0.45	+1.74	−0.30	−1.13
		25.86				
Mean ordinate of all the Carcel lamp curves = 28.27.						
					Methven screen curves.	
CURVE XIX.	35–40	23.87	−0.91	−3.67	−2.74	−10.30
	40–45	24.89	+0.11	+0.44	−1.72	−6.47
	45–50	24.98	+0.20	+0.81	−1.63	−6.13
	50–55	25.38	+0.60	+2.42	−1.23	−4.63
		24.78				
CURVE XX.	60–65	25.31	−1.07	−4.06	−1.30	−4.88
	65–70	25.82	−0.56	−2.12	−0.79	−2.97
	70–75	26.24	−0.14	−0.53	−0.37	−1.39
	75–80	26.59	+0.21	+0.80	−0.02	−0.08
	80–85	26.82	+0.44	+1.67	+0.21	+0.79
	85–90	27.48	+1.10	+4.17	+0.87	+3.27
		26.38				
CURVE XXI.	0–5	28.10	−0.63	−2.19	+1.49	+5.60
	5–10	28.97	+0.24	+0.84	+2.36	+8.87
	10–15	28.81	+0.08	+0.28	+2.20	+8.27
	15–20	29.06	+0.33	+1.15	+2.45	+9.27
		28.73				
CURVE XXII.	25–30	28.87	−0.16	−0.55	+2.26	+8.51
	30–35	29.19	+0.16	+0.55	+2.58	+9.70
		29.03				
Mean ordinate of all the Methven screen curves = 26.61.						

IV.

Discussion of the Results Obtained.

Plate I. exhibits the curves given by the English standard candles. In curves III. and IV. the candles were lighted at their tops. The wicks flared up and gave a high point on each curve, which in the case of curve IV. is not shown on the plate. The flames then increased gradually to their normal size, which was reached after about 15 minutes. Curves III. (*a*) and IV. (*a*) are continuations of III. and IV. The candles were allowed to burn during the interval between the curves, which in the case of III. and III. (*a*) was 45 minutes, and in the case of IV. and IV. (*a*) was 55 minutes. During III. the room was rather more draughty than during III. (*a*), and the effect of the draughts is seen in the much larger numbers of small irregularities in the former than in the latter curve. During the interval between IV. and IV. (*a*) the height of the flame of the candle was found to vary between 43 and 48 mm.

Curve I. was taken with the portion of candle left over from IV. and IV. (*a*). It was lighted, its wick being already charred and its crater formed, and readings were taken immediately. Curve II. was taken with the lower half of the candle used in getting III. and III. (*a*). The bottom, *i.e.* the larger end of this, was hollowed out to expose the wick and readings were taken after the candle had been burning long enough to come to its normal light-giving power. The agreement in the amount of radiation of the candle burned in this way, with the amount when burned from the smaller end, shows that the variation in the diameter of the candle has little if any influence on the intensity of the light emitted.

One marked peculiarity which characterizes, to a greater or less degree, all these curves, is the succession of sudden drops followed by gradual rises to a maximum. In the case of the drop in curve IV. at 55 minutes, the change amounted to 15 per cent of the total deflection, and in other instances the change was nearly or quite as large. The reason for these drops is to be looked for in the action of the wick, which, as the candle burns

down, projects farther above the spermaceti, causing a tall flame. Finally, by reason of charring and because of its own weight, it bends over, and the end burns off. The flame, following the wick, becomes shorter and, sometimes, broader. Since the wicks in the English standard candles are very uniform in construction, these drops succeeded each other after more or less regular time intervals, which were, in most cases, not far from three minutes. The curves show also a comparative freedom of these flames from minor fluctuations due to disturbing draughts of air. The ordinates of the English candle curves are not corrected for rate of burning. It will be seen from the tables that these corrections, while they have in each case the proper sign, are, in most cases, entirely insufficient to reduce the curves to a common mean ordinate.

Plate II. exhibits the curves of German standard paraffin candles as prescribed by the *Deutscher Verein von Gas- und Wasserfachmännern*, and called the "Vereinskerze."

Curves V. and V. (*a*) and VI. and VI. (*a*) were obtained from candles burned in exactly the same way as III. and III. (*a*) and IV. and IV. (*a*), the time interval between V. and V. (*a*) being one hour, and that between VI. and VI. (*a*) being 41 minutes. Curve V. (*b*) was taken with the remainder of the candle used for V., relighted on another night; so that it is entirely independent of V. and V. (*a*), excepting that the same candle was used.

The general level of these curves is rather closely adhered to. The numerous minor fluctuations show that the flame of this candle is quite subject to disturbances by draughts. Occasionally the flame increases to a size far beyond its normal, and smokes. These large fluctuations, extending over a considerable period of time, form a marked feature of the curves. It will be noticed that VI. did not reach its normal light-giving power until 15 minutes later than V.

When we come to compare the English with the German candle, we notice, first, that the variations of the English candle were much larger and that large variations were much more frequent. The German curves are free from the semi-periodic drops which

characterize the English. Curve I. at 24 minutes reaches a value which is 1.235 of the mean ordinate of the English candle curves. Curve IV. at 49 minutes drops to a point which is only 0.77 of the mean ordinate. The total variation is, consequently, 46.5 per cent. Both of these curves were taken with the same candle, but on different nights. The highest point of the German curves is on V. (*a*) at 86 minutes. The ordinate reaches a value which is 1.155 of the mean. The lowest point is on VI. (*a*) at 72 minutes, and is 0.915 of the mean. The total variation was consequently 24 per cent, or only about half that shown by the English candle.

Moreover, we see from the table that the percentage deviation of curve III., 55–60 minutes, from the mean of the English candles is + 10.45 per cent. Curve IV., 35–40 minutes, shows a deviation of − 13.08 per cent. The total deviation for a period of five minutes is 23.53 per cent for the English candles. In the German candles the maximum positive and negative deviations for five-minute periods are + 11.52 per cent and − 10.43 per cent respectively. These give a total of 21.95 per cent, and exhibit a performance but little better than that shown by the English candle.

These figures make perfectly evident the futility of any attempt to get concordant photometric results from freely burning candles, unless, indeed, a very long series of observations is taken. In favor of the English candle it can be said that its flame is more stable and is much less subject to smaller fluctuations than is the German candle. When it comes to measuring the dimensions of the flame and reducing readings to a standard size of flame, there is little doubt that the English candle would be found much superior to the German. In the early part of the work with candles a rather rude measuring device was set up and heights of flame of the English candle were determined and compared with the corresponding galvanometer readings. The accordance between definite flame heights and definite galvanometer readings was found to be very close.

Investigations along this line have not been prosecuted farther for lack of another observer. A device has been made, however, by which it is hoped that the height, or the height and the width, of a candle flame can be measured with great quickness and accu-

racy, and it is the purpose of one of the present observers to attempt to ascertain, by means of this measuring apparatus and the bolometer, whether the radiating power of a candle can be expressed conveniently in terms of its simpler dimensions or not. That the light-giving power of a candle must vary directly with the cubical contents of the luminous portion of its flame, provided the surrounding conditions are unaltered, seems obvious. That the relative luminous intensities of candles of different materials can be determined with a fair degree of accuracy by measuring the cubical contents of the light-giving portion of their flames has recently been shown by Glan.[1] For photometric work, however, it is essential that the luminous intensity at any time should be deducible from simple measurements of the flame — such measurements as can be made the instant before or after a photometer setting is made.

Plate III. shows the behavior of the Hepner-Alteneck amyl-acetate lamp. The lamp used is one by Hartmann and Braun, of the type having the optical projection device for fixing the height of flame at 40 mm.

Curve VII. was taken when a window in the farthest corner of the room was raised about 2 cm. Curve VII. (*a*) was begun 25 minutes after the end of VII., the lamp having burned during the interval, and the flame having been readjusted in height. The window was closed. The marked difference between the two curves is due to the stoppage of this slight draught.

In taking curve X., the sensitiveness was adjusted at 32.5, or about double the standard sensitiveness for the other tests of the candles and of the Hefner lamp. Then a computation was made, from the law of inverse squares, of the distance at which it would be necessary to place the lamp in order to produce the ordinary deflection. The fact that the ordinates of this curve agree with those of the curves taken with a sensitiveness of 16.2 shows that the method of getting the sensitiveness by taking only first throws of the galvanometer needles, was an allowable one.

Curves XI., XI. (*a*), XI. (*b*), XI. (*c*) show the results of successive attempts to adjust the height of flame to just 40 mm., the wick

[1] P. Glan, Ueber ein Gesetz der Kerzenflammen, Wied. Ann., Vol. 51, p. 584.

having been lowered between each one. No zero and sensitiveness readings were taken between the beginning of XI. and the end of XI. (*c*).

If the times that the lamp had been burning, when curves VIII., XI., XII., and XIV. were begun, are taken from Table I., it will be seen that, in each case, it was 20 minutes or less. Each of these curves shows a gradual increase in the amount of radiation. This emphasizes the advisability of lighting the Hefner lamp a considerable time before it is to be used in photometric work.

It is to be understood that, in the case of each of these runs except the one when curve VIII. was taken, the greatest care was exercised to have the room as quiet as possible. Curve VII. was the first of the Hefner lamp curves taken, and the need of extraordinary care in excluding draughts of air was not appreciated.

The Hefner light is a much more promising standard than the candles, although its curves show that it is subject to incessant fluctuations and that considerable variations may last for a minute or more. It is clear, however, that from a series of observations, extending over a comparatively short time, a very good average may be expected.

The highest point on the Hefner curves is 1.093 of the mean, and the lowest is 0.867, giving a total deviation of 22.6 per cent. This large deviation is not so significant, however, as a similar one in the case of candles, since in dealing with the Hefner lamp we have to do with an adjustable flame, and these fluctuations may indicate only that a slight readjustment was needed.

The most important question in regard to the Hefner lamp is the accuracy with which it can be adjusted to its normal light intensity. Some idea may be formed of this by noting in Table II. the percentage deviations of the first five minutes of each of the curves from the mean ordinate of the first five minutes of all the curves. The maximum deviation will be seen to be 8 per cent, while the mean deviation is 2.3 per cent. To show the accuracy with which the flame can be adjusted under given unchanging conditions, curves XI., XI. (*a*), (*b*), and (*c*) are of especial value.

In Table III. will be found the mean ordinates for the first five minute of XI. and of all of XI. (*a*), (*b*), and (*c*), and the percentage deviation of each from the mean of all.

TABLE III.

Curve.	Mean ordinate.	Deviation from mean.	Percentage deviation.
XI. (5^m)	39.42	−0.14	−0.35
XI. (*a*)	39.37	−0.19	−0.48
XI. (*b*)	40.00	+0.44	+1.11
XI. (*c*)	39.46	−0.10	−0.25
Mean	39.56	Mean	0.57

If, now, the assumption be made that the light-giving efficiencies of the English and German candles and of the Hefner lamp are equal, it is possible to get their relative intensities by a comparison of the mean ordinates of their curves. Reducing the deflections as read on the telescope scale to angular measure and taking double the tangents of these angles, we have for the true deflections the following values: —

$$\text{English candle} = 41.06$$
$$\text{German candle} = 50.40$$
$$\text{Hefner light} = 38.66$$

From these values we get the following ratios: —

$$\frac{\text{German candle}}{\text{English candle}} = 1.2275$$

$$\frac{\text{Hefner light}}{\text{English candle}} = 0.9415$$

Violle's values for these ratios are 1.13 and 0.98 respectively.

The curves given by the Carcel lamp are numbered XV., XVI., XVI. (*a*), XVII., and XVII. (*a*). They are all reduced to a common rate of burning of 42 grams per hour. XV. was begun when the lamp had been burning 38 minutes. It shows a slight upward tendency which decreased as time went on. The rate of consumption was high, — 48 grams per hour. XVI. was begun 35 minutes after the lighting of the lamp, and the rate of burning was 43 grams. XVI. (*a*) shows the radiation from the lamp after it had

been burning nearly two hours. During XVI. the radiation was increasing throughout the test. After the lamp had been burning during the interval between XVI. and XVI. (*a*), the radiation was falling off very rapidly as the wick became more and more charred. The rate for XVI. (*a*) was higher than for XVI., having been altered by a change in height of the chimney.

XVII. was begun 21 minutes after the lamp was lighted, while XVII. (*a*) was not begun until the lamp had been burning three hours. XVII. shows a rise as the lamp became warmed, the curve becoming parallel to the base line when the lamp had been burning half an hour. XVII. (*a*), however, shows that the radiation was again increasing. During XVII. the rate was 46.8 grams per hour, while the rate during XVII. (*a*) was only 40 grams per hour.

These curves show clearly the great gain in steadiness of a flame resulting from the use of a proper chimney. The deviations were, for the most part, no larger than the swings of the galvanometer when the bolometer was unexposed to radiation. The variations extending over a considerable period of time are, however, by no means inconsequential. In curve XVII. there was a total deviation in the mean ordinates of the five-minute periods of 5.1 per cent, taking place in 10 minutes. Curve XVI. shows a deviation of less than 0.8 per cent in 35 minutes, and this is perhaps a more typical illustration of what may be expected from this lamp when burned under the best conditions. The highest and lowest points on any of the Carcel curves are 29.9 and 25.4 respectively, corresponding to deviations from the mean of the curves of + 5.7 per cent and − 12.5 per cent, or a total of 18.2 per cent. Table II. shows the maximum deviations, for periods of five minutes, to be + 5.3 per cent and − 8.4, giving a total of 13.7 per cent.

Table IV. serves to show the reproducibility of the standard at different times.

TABLE IV.

Curve.	Mean ordinate.	Deviation from mean.	Percentage deviation.
XV.	27.60	−0.50	+1.78
XVI.	29.37	+1.27	+4.52
XVI. (*a*)	28.63	+0.53	+1.89
XVII.	26.82	−1.28	−4.56
XVII. (*a*)	28.06	−0.04	−0.14
Mean	28.10	Mean	2.58

The mechanical parts of the lamp used were in good working order. Pure Colza oil was used for burning, and rates of combustion were determined by weighings made before and after the runs. The regulation wicks were employed, a fresh one being used for each run.

The curves given by the Methven screen are numbered XVIII., XIX., XX., XXI., and XXII. Curve XVIII. was taken with the Argand burner connected directly to the gas-pipes in the building. The pressure of the gas was controlled only by a large regulator on the main pipe leading into the building. The consumption of gas in the building was constant during the run. The curve shows many large, but quick, variations and certain decided waves. The other Methven curves were taken with a gas-holder of about 10 gallons' capacity interposed between the gas main and the Argand burner. The effect of this in smoothing the curves is very marked, and indicates that for photometric purposes the Methven screen is much improved by the interposition of a rather capacious reservoir. The use of such a reservoir tends to minimize the effect of changes of pressure and to absorb any waves in the gas due to water in the pipes or some similar cause.

The way in which these waves of variable pressure produce fluctuations in the amount of radiation is by causing changes in the quality of those portions of the flame which cover the slit. At times the top of the flame becomes forked, so that not all of the slit is covered by it. It happens perhaps more frequently

that some of the non-luminous portion of the flame rises so as partially to cover the slit. In either case the result is seen in a deviation of the curve. It must be true, also, that the amount of luminous radiation suffers a much larger proportional change than the total amount of radiation; hence the deviations recorded on the curves are too small to represent correctly the fluctuations in luminous intensity. There is an additional reason for these fluctuations, which is to be found in the slight protection against draughts of air which the wide open chimney used on the London Argand burner affords. In this respect its action is in marked contrast with that of the smaller close chimney of the Carcel lamp, which, of course, can be used only with much richer gases than ordinary illuminating gas.

These Methven screen curves cannot be trusted to give a true idea of the reproducibility of the standard. This is because the ordinates of the curve represent not only the radiation from the flame, but also that from the chimney and from the screens interposed. Since the burner was not always replaced in exactly similar positions before the bolometer, the amount of this radiation was variable. A more serious source of error, however, arose from the fact that after several of the curves had been taken it was found advisable to change the original arrangements for screening. The plan finally followed was to interpose between the Methven screen and the bolometer-box a heavy piece of asbestos paper, having a hole of proper size cut in it. This was placed so far from the burner that it did not become heated perceptibly.

The difficulty arising from the intermingling with the radiation from the flame of a variable amount of non-luminous radiation from chimney and screens could have been entirely obviated by determining, in a way which did not suggest itself until after most of the runs had been taken, the effect of this variable radiation. After the burner had been lighted long enough for the chimney and screens to become heated to their ultimate temperature, the flame was suddenly extinguished and the screen covering the bolometer strip was raised. Then a curve of cooling of the hot chimney and screens was plotted, using times as abscissas and deflections as ordinates. This curve carried back to the time of extinguishing the

light gave the amount of radiation received from all parts except the flame. The importance of this correction was not fully appreciated until the results of observations were worked over, after all the runs had been taken; consequently the method was not developed very completely. The records show that in the case of the Carcel lamp the amount of radiation from the chimney varied slightly. At the end of curve XV. the chimney radiation was such as to produce a deflection of 6.2 scale divisions, while at the end of XVII. (*a*) the chimney radiation was 5.8. For curve XVI. no observations of chimney radiation were made; but since the conditions under which the run was taken were very similar, the amount of such radiation could not have differed greatly from that found for the other curves. That the effect of applying this correction is not necessarily to tend to reduce the curves to a common mean ordinate is indicated by the fact that if we correct the end of XV. by subtracting from it 6.2, and correct the end of XVII. (*a*) by subtracting from it 5.8, the difference between these portions of the curves will be exaggerated.

The only determinations of the radiation from the chimney and the intervening screens in the case of the Methven standard were made at the end of curves XXI. and XXII. For curve XXI. this radiation caused a deflection of 2.8 divisions, and for XXII. the deflection was 2.7 divisions. These curves were taken on the same night, and only a short time intervened between them. The apparatus was not displaced in the meantime.

The above work was done during the fall and winter of 1893–4. We have, in conclusion, to express our earnest thanks to Professor E. L. Nichols, not only for having proposed the line of research, and having outlined the method to be employed, but also for many valuable suggestions given during the progress of the investigation.

CORNELL UNIVERSITY, April, 1894.

ON A RELATION BETWEEN SPECIFIC INDUCTIVE CAPACITY AND CHEMICAL CONSTITUTION OF DIELECTRICS.

By Charles B. Thwing, Ph.D.

THAT a relation between specific capacity and chemical constitution of dielectrics has not up to this time been found, is no doubt due to the small number of substances hitherto determined which were adapted to show such a relation. That I have been able to determine, with a good degree of accuracy, a very large number of substances, and so to find, as I believe I have done, such a relation, is due to the extreme simplicity of the method employed for determining capacities.

For the ground idea of this method, the determination of capacity by means of resonance, I am indebted to Professor Hertz, by whose untimely death I was deprived, almost at the beginning of the work, of that friendly counsel and encouragement which he was always so willing to bestow.

The principle may be stated in the following form: since the period of the electrical oscillations which occur at the discharge of an electrical system depends only upon the capacity and self-induction of the system, it follows that, if two systems, A_1, A_2, have the same self-induction, their capacities, C_1, C_2, will be equal when both systems are in resonance with a third system, A_3, whatever the self-induction and capacity of the latter may be.

A convenient method of employing this principle for the measurement of capacities requires the following apparatus: (1) a primary circuit with variable air-condenser and an induction coil of moderate dimensions; (2) a secondary circuit with variable air-condenser which may be replaced by the condenser whose capacity is to be measured; (3) a convenient means of measuring the amount of current developed in the secondary circuit as the relative capacities of the two circuits is varied.

For the latter purpose a dynamometer invented by Professor Hertz[1] was found to be, with some modifications, very well adapted. The instrument is shown in Fig. 1. A german silver wire, *ab*, 60 cm. long and 0.12 mm. in diameter, is divided at its middle point, and the two portions soldered to opposite sides of a short steel wire, *cd*, 0.86 mm. in diameter. To each end of the thick wire, *cd*, is soldered a thin steel wire the free ends of which are attached, the lower to a fixed support at *e*, the upper to a torsion head, *T*, by means of which the long wire, *ab*, is kept constantly taut. The passage of a current through *ab* causes it to expand, allowing the wire, *cd*, to turn upon its axis in the direction of the torsion. A mirror, *m*, is attached to *cd*, the deflections of which are observed by means of a lamp and scale placed at a distance of about 2 m. from *m*. A lens of about 2 m. focal length, placed just in front of *m*, gives a sharp image of the flame upon the scale.

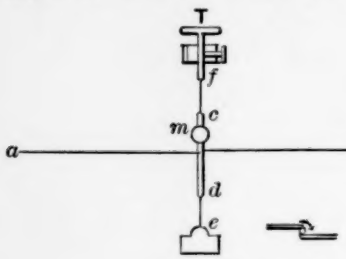


Fig. 1.

With interrupted currents the alternate heating and cooling of the wire produces small oscillations in the mirror, which, while blurring the image to such an extent as to exclude the use of a reading telescope, are not sufficiently great to prevent accurate readings with the lamp and scale. The current obtained by connecting one Planté element in the dynamometer circuit gave a deflection of 250 scale divisions. The deflection ordinarily obtained in making determinations varied from 100 to 250.

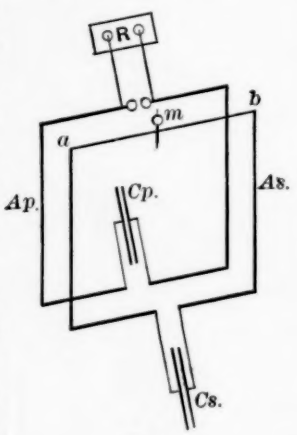


Fig. 2.

The arrangement of the apparatus as a whole is shown in Fig. 2. The primary circuit, A_p, is a quadrangle of copper wire 60 cm. square, the wire being 1 mm.

[1] Zeitschrift für Instrumentenkunde, III. p. 17, 1883.

in diameter. The secondary circuit, A_s, has the same dimensions and is placed 15 cm. distant from A_p with its sides parallel to it. The side of A_s which lies opposite the spark gap in A_p is replaced by the dynamometer.

The primary condenser, C_p, Fig. 3, consists of two circular plates of zinc, each 40 cm. in diameter, one of which is attached to a fixed support, the other to a support which moves smoothly upon two stout glass tubes, R, R, and is provided with a pointer, z, and millimeter scale, m. As employed in this research it was not essential that the plates be exactly parallel; it was, however,

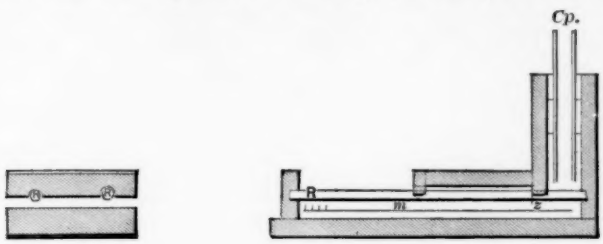


Fig. 3.

necessary that they remain always in the same relative position for the same scale reading.

The secondary condenser, C_s, was a Kohlrausch condenser, with plates of 15 cm. diameter, and was provided with adjusting screws and a scale and vernier reading to tenths of a millimeter.

A table was computed for this condenser for all values of a from 0.1 mm. to 30 mm. by Kirchhoff's[1] formula:

$$C=\frac{R^2}{4a}+\frac{R}{2\pi}\left(\log\frac{8\pi R}{a}-1\right),$$

where R is the radius and a the distance apart, of the plates. The values run from 149.59 at 0.1 mm. to 8.49 at 30 mm. To save time, the capacity of C_s corresponding to each position of C_p was determined once for all. With C_p set, for example, at 73 mm., C_s was adjusted to show a maximum scale reading, the distance a, read and recorded, and the observation repeated. Table I. shows a series of readings taken at random, and will serve as an indication of the accuracy of the observations in general.

[1] Kirchhoff, Electricität, p. 109.

TABLE I.

a_s	80	84	89	84	80	89	90	Mean 85
C_s	23.07	22.35	21.19	22.35	23.07	21.19	20.95	22.12

In order to obtain these values, which enter into all following observations, with the greatest possible accuracy, the individual values were plotted in a curve, and a mean thus obtained of the series.

Table II., and the corresponding curve in Fig. 4, shows the sensitiveness with which a maximum deflection is indicated. The first line gives the distances, the second the deflections with

TABLE II.

Distance.	35	40	45	50	55	60	62	64
Deflection.	80	102	132	150	165	182	195	200
Distance.	66	70	75	80	90	100	110	130
Deflection.	195	181	162	141	105	86	70	50

condenser III., containing turpentine as dielectric, in the secondary circuit.

In the case of substances with a perceptible conductivity, the deflection was smaller. No determination was attempted when the deflection fell below 30 scale divisions. It is, however, to be borne in mind that the conductivity of the dielectric is independent of its capacity; it diminishes, nevertheless, the exactness of the determination by decreasing the magnitude of the quantity we are to measure, — the deflection of the dynamometer.

Owing to the proximity of the secondary circuit, the primary circuit is noticeably affected by changes in the secondary. When the current breaker is well adjusted, the approach of the secondary

circuit to the condition of resonance is often indicated by a change in the note emitted by the current breaker. The vibrations become more regular as resonance is approached, and again irregular as soon as the point of maximum effect is passed. An instrument that should respond with sufficient certainty to this reaction of the secondary circuit upon the primary might be made a very delicate means of detecting resonance.

It remains to describe the condensers used for determining specific capacities. Solids were examined in plate form clamped

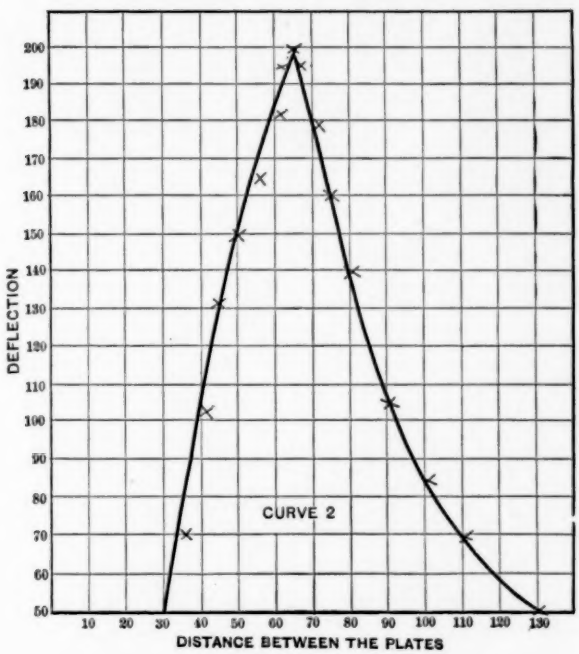


Fig. 4.

between two circular plates of metal varying in diameter from 3 to 12 cm., the capacities being computed by Kirchhoff's formula given above. The specific inductive capacity, K, of the substance is, of course, equal to the ratio between the capacity of the condenser with the given dielectric between the plates and the capacity of the same condenser in air as computed by the formula.

The first determinations of liquids were made with a condenser consisting of two balls supported at the middle of a glass cylinder upon wires admitted through the sides of the cylinder. The disadvantage of this plan was the large amount of liquid required. It was therefore abandoned for the form shown in Fig. 5. The liquid is here contained between that portion of two concentric cylinders in which the field of force is uniform, the space at the ends of the tube being filled with ebonite plugs which are screwed into the cylinders to a depth of 5 mm. The stopper, *s*, which fits flush with the inside of the cylinder, is hollow and has an opening in the bottom 1 mm. in diameter to allow the surplus liquid to escape. In this form the condenser offers no exposed liquid surface for the conduction of an air discharge between the two cylinders.

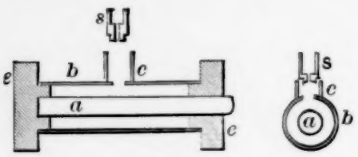


Fig. 5.

The capacity C_e of the ends is determined by measuring the capacity of the whole condenser in air and deducting the computed capacity of the central portion. The latter is computed by Maxwell's [1] formula:

$$C=\frac{1}{2}\,\frac{l}{\log\frac{b}{a}},$$

where l is the length of the cylinders, b the radius of the outer, and a of the inner cylinder.

We then have:

$$K=\frac{C'-C_e}{C}.$$

For the determination of substances having widely varying specific capacities three condensers of the dimensions shown in Table III. were employed. It will be seen that most substances could be measured by two of these condensers. The amount of liquid required, as shown in column 3, was never more than 3.5 cc., — a very important item when a large number of substances are to be examined.

[1] Maxwell, Electricity and Magnetism, Vol. I. p. 177.

TABLE III.

No.	l	Volume.	b	a	C	C_0	For measuring capacities.
I.	1	1.55	7.15	1.25	0.287	8.0	From $K = 15$ to $K = 100$
II.	2	2.45	7.15	3.48	1.388	10.8	From $K = 5$ to $K = 25$
III.	4	3.37	7.15	5.04	5.5998	11.5	From $K = 1$ to $K = 15$

Owing to the difficulty of obtaining many solid substances in the form of plates, I was led to adapt the cylindrical condensers to the measurement of solids as well as liquids. The condenser was first weighed, then filled with the solid substance in the form of powder, and again weighed. The space between the solid particles was then filled with a liquid in which the solid is insoluble, usually alcohol or ether. The volume of the substance is computed from its known specific weight, and this volume subtracted from the volume of the condenser gives the volume of the liquid. The specific capacity of the mixture is equal to the volume of the liquid times its capacity plus the volume of the solid times its capacity, divided by the total volume of the mixture; or,

$$K_m = \frac{V_l K_l + V_s K_s}{V_m}$$

whence,

$$K_s = \frac{V_m K_m - V_l K_l}{V_s}.$$

The Results.

The results obtained are shown in the following tables. Those given in the first half of Table IV. were obtained with the solid substance in the form of plates; those in the second half were obtained by means of the cylindrical condensers.

The first value for ice is entitled to less weight than the second, since the temperature of the room was 0°, so that the formation of a film of water on the plates was not impossible. In the second case the condenser was immersed in an oil bath which was kept for half an hour at a temperature of $-5°$.

The determinations were made in all cases where the temperature is not given, at a temperature of 15° C.

The value given for sugar in liquid form was obtained by extrapolation from three aqueous and two alcoholic solutions. Honey, a substance of indefinite chemical constitution, but containing always some cane sugar, was found to have a specific capacity of 35. Tartaric acid was determined from an alcoholic solution.

TABLE IV.

SOLIDS.

Substance.	Plates.		K	Substance.	Mixture in cylinder.	K
	a	R				
Paraffine	0.42	4.05	2.14	Ice, −5°		2.85
	0.66	4.05	2.18	Cetylalcohol		6.42
Ebonite	0.42	4.05	2.71	Acetophenone		16.24
Wood, oak	2.64	6.00	2.93	Glacial acetic acid		2.79
" pine	1.58	6.00	2.95	Sand	Ether	7.43
Paper	0.6	4.05	2.82	Potassium chlorate	"	6.18
Porcelain	0.8	4.05	4.20	" carbonate	"	5.62
Glass (mirror)	0.1	4.05	5.84	" sulphate	"	6.45
Sandstone	2.8	4.05	6.20	Copper sulphate	"	5.46
Marble (white)	1.93	4.05	6.13	" oxide	Alcohol	18.10
" (black)	1.86	4.05	6.15	Iron oxide	Ether	14.20
Iceland spar	1.4	2.0	7.40	Barium nitrate	"	9.15
	1.4	2.5	7.34	" sulphate	"	11.40
Rock salt	0.68	2.0	5.81	Lead carbonate	"	18.58
Gypsum	0.54	2.0	5.61	" oxide	Alcohol	25.90
Ice, −2°	1.50	4.05	3.36	" sulphide	"	17.92
Cane sugar	1.0	2.0	4.19	Cane sugar	Ether	4.13

It seemed to me a matter of especial interest to examine a series of mixtures of alcohol and water in various proportions, as well as similar mixtures of other substances which mix readily with water in all proportions. The results are shown in Tables VI. to XI., and in the curves with corresponding numbers shown in Figs. 6 and 7. For mixtures of ethyl and methyl alcohol the value lies always under the mean value for the two bodies in a free state. For all the aqueous mixtures examined, each of the

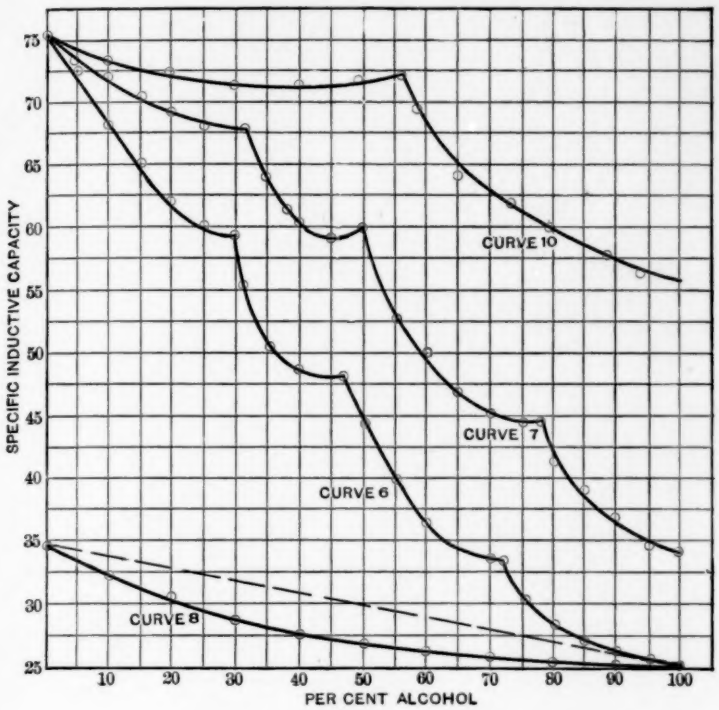

SPECIFIC INDUCTIVE CAPACITY
75
70
65
60
55
50
45
40
35
30
25
CURVE 10
CURVE 7
CURVE 6
CURVE 8
10
20
30
40
50
60
70
80
90
100
PER CENT ALCOHOL

Fig. 6.

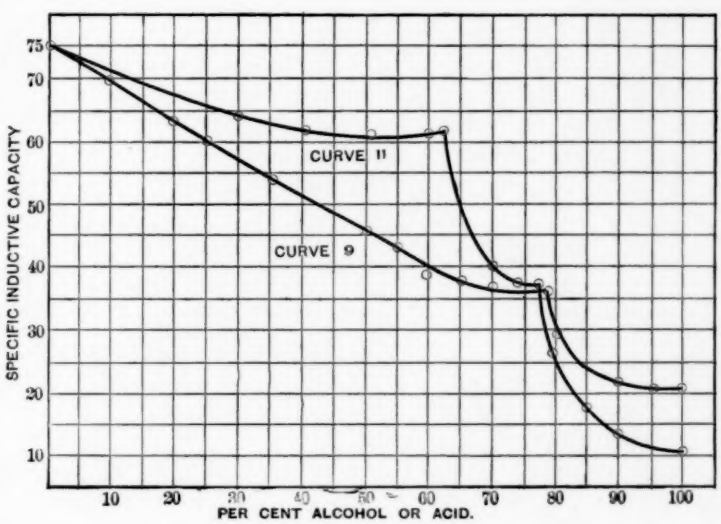

SPECIFIC INDUCTIVE CAPACITY
75
70
60
50
40
30
20
10
CURVE 11
CURVE 9
10
20
30
40
50
60
70
80
90
100
PER CENT ALCOHOL OR ACID.

Fig. 7.

TABLE V.

LIQUIDS.

Substance.	K	Substance.	K
Water	75.50	Aldehyde	18.55
Glycerine	56.20	Salicylaldehyde	19.21
Cane sugar (in water)	52.00	Propylaldehyde	14.41
" " (in alcohol)	55.00	Benzaldehyde	14.48
Methylalcohol	34.05	Valeraldehyde	11.76
Ethylalcohol	25.02	Cuminaldehyde	10.68
Propylalcohol	20.45	Chloral	5.47
Isopropylalcohol	19.82	Acetone	21.85
Amylalcohol	14.62	Methylethylketone	18.44
Allylalcohol	21.60	Acetylchloride	25.30
Tartaric acid	35.90	Methylpropylketone	16.75
Formic acid	62.00	Acetophenone	16.24
Lactic acid	20.90	Dipropylketone	12.44
Acetic acid	10.30	Methylhexylketone	10.42
Propionic acid	5.50	Nitrobenzole	32.19
Butyric acid	3.16	Nitromethane	56.36
Valeric acid	3.06	Nitrotoluole	26.58
Creosol	11.75	Ethylnitrate	17.72
Toluole	2.37	Ethylether	4.27
Turpentine	2.28	Chloroform	3.95
Carbon disulphide	2.50	Bromoform	7.42

curves shows one or more singular points, which fall, without exception, upon such percentages as correspond to a chemical combination of the substance with water.

In the case of propylalcohol and of acetic acid such hydrates are already known. The remaining substances give evidence of chemical union, either by absorbing water vigorously from the air or by contraction or the evolution of heat on mixing.

It is to be remarked, with reference to acetic acid, that, owing to the increase in conductivity on the addition of water, no exact determinations could be made of mixtures containing less than 30 per cent acid.

TABLE VI.

WATER AND ETHYLALCOHOL.

Per cent alcohol.	K	Corresponding hydrate.	Per cent alcohol.	K	Corresponding hydrate.
0.0	75.50		55.0	39.93	
5.0	72.30		60.0	36.31	
10.0	67.95		65.0	34.60	
15.0	65.36		70.0	33.66	
20.0	61.79		72.0	33.86	$C_2H_6O + H_2O$
25.0	60.21		75.0	30.30	
30.0	59.55	$C_2H_6O + 6\,H_2O$	80.0	28.15	
32.0	55.20		85.0	26.58	
35.0	50.52		90.0	25.71	
40.0	48.40		95.0	25.27	
46.0	48.40	$C_2H_6O + 3\,H_2O$	99.8	25.02	
50.0	44.11				

TABLE VII.

WATER AND METHYLALCOHOL.

Per cent methyl-alcohol.	K	Corresponding hydrate.	Per cent methyl-alcohol.	K	Corresponding hydrate.
0.0	75.50		50.0	57.14	
5.0	73.24		55.0	52.51	
10.0	72.30		60.0	50.00	
15.0	70.73		65.0	46.76	
20.0	69.20		70.0	45.16	
25.0	67.94		75.0	44.11	
30.8	67.94	$CH_4O + 4\,H_2O$	78.0	44.62	$2\,CH_4O + H_2O$
35.0	64.14		80.0	40.98	
37.0	61.50		85.0	38.05	
40.0	60.22		90.0	36.30	
45.0	59.41		95.0	34.05	
47.5	60.22	$CH_4O + 2\,H_2O$	99.7	34.05	

TABLE VIII.

METHYLALCOHOL AND ETHYLALCOHOL.

Per cent ethylalcohol.	*K*	Per cent ethylalcohol.	*K*
0.0	34.05	60.0	26.20
10.0	31.91	70.0	25.71
20.0	30.21	80.0	25.34
30.0	28.54	90.0	25.36
40.0	27.59	100.0	25.02
50.0	26.78		

TABLE IX.

WATER AND PROPYLALCOHOL.

Per cent propyl-alcohol.	*K*	Hydrate.	Per cent propyl-alcohol.	*K*	Hydrate.
0.0	75.50		65.0	37.52	
10.0	69.50		70.0	36.87	
20.0	62.80		77.0	36.00	$C_3H_8O + H_2O$
25.0	60.08		80.0	28.92	
35.0	53.35		90.0	21.60	
50.0	44.63		95.0	20.73	
55.0	42.00		99.8	20.45	
60.0	38.75				

TABLE X.

WATER AND GLYCERINE.

Per cent glycerine.	K	Hydrate.	Per cent glycerine.	*K*	Hydrate.
0.0	75.50		58.0	69.20	
10.0	73.24		65.0	64.14	
20.0	72.30		73.0	61.79	
30.0	71.36		80.0	59.40	
40.0	71.36		89.0	57.80	
48.5	71.46		94.0	56.54	
56.0	72.30	$C_3H_8O_3 + H_2O$			

TABLE XI.

WATER AND ACETIC ACID.

Per cent acetic acid.	K	Hydrate.	Per cent acetic acid.	K	Hydrate.
0.0	75.50		70.0	39.48	
30.0	64.20		74.0	36.62	
40.0	61.79		77.0	36.87	$C_2H_4O_2 + H_2O$
50.0	60.80		80.0	22.90	
60.0	60.80		100.0	10.30	
62.5	61.25	$C_2H_4O_2+2\,H_2O$			

Table XII., and the corresponding curve in Fig. 8, shows the influence of temperature upon the specific capacity of water, concerning which I will speak again farther on.

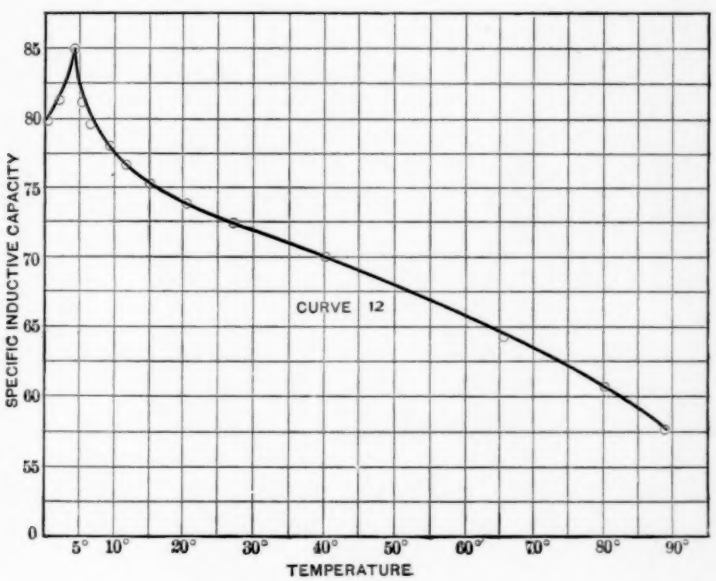


Fig. 8.

TABLE XII.

WATER.

Temperature.	K	Temperature.	K
0°	79.46	15°	75.50
2°	80.84	20°	73.92
4°	85.20	27°	72.70
6°	80.84	40°	69.80
7°	79.40	65°	64.32
9°	77.95	80°	60.50
12°	76.20	88°	57.90

The evident connection between the dielectric constants of the alcohol series led me to seek a relation which might be expressed as a function of the chemical constitution of the dielectric. The variations produced by temperature, and the high value found for sugar and glycerine, as well as the difference between the two propylalcohols, made it probable that the specific capacity varies directly with the density. It was further clearly noticeable that the members of the series containing a large proportion of the atom group hydroxyl had relatively large valves.

By way of trial I assumed the following formula: —

$$K=\frac{D}{M}(a_1K_1+a_2K_2+\cdots),$$

where D is the density, and M the molecular weight of the substance, K_1, K_2, etc., constants for each element or atom group, and a_1, a_2, etc., the number of atoms of the element in the molecule. I determined first the values for hydrogen, K_H, and carbon, K_C, from such substances as contained only those elements. The values were so nearly in the ratio of the atomic weights of the substances that that ratio was adopted. Calling, then, $K_H=2.6$, $K_C=31.2$, that value for hydroxyl ($K_{(OH)}=1356$) was taken, which satisfies the equation for water. The closeness with which the equation was satisfied for all the other liquids containing the hydroxyl group was so remarkable that I proceeded to examine the aldehydes, acetones, etc. The computed and observed values are given in parallel columns in Table XIII.

Table XIII.

$$K = \frac{D}{M}(a_1K_1 + a_2K_2 + \cdots)$$

$K_H = 2.6$	$K_{(OH)} = 1356$	$K_{(CH_2)} = 41.6$
$K_C = 2.6 \times 12$	$K_{(CO)} = 1520$	$K_{(CH_3)} = 46.8$
$K_O = 2.6 \times 16$	$K_{(COH)} = 970$	$K_S = 2.6 \times 32/2$
$K_X = 2.6 \times M_X$	$K_{(NO_2)} = 3090$	

Substance.	Formula.	D	M	K computed.	K observed.	Observer.
Water	$H.OH$	1.0	18.0	75.50	75.50	Thwing
					75.70	Rosa
					76.00	Cohn & Arons
					73.50	Cohn
					83.70	Tereschin
					69.00	Yule
Glycerine	$C_3H_5(OH)_3$	1.26	92.0	57.17	56.20	Thwing
Cane sugar	$C_{12}H_{14}(OH)_8O_3$	1.588	342.0	53.67	53.50	"
Methylalcohol	$CH_3.OH$	0.796	32.0	34.38	34.05	"
					32.60	Tereschin
Ethylalcohol	$C_2H_5.OH$	0.8	46.0	25.00	25.02	Thwing
					26.20	Mean: 5 observers
Propylalcohol	$CH_3(CH_2)_2.OH$	0.82	60.0	20.32	20.45	Thwing
-iso		0.7879	60.0	19.51	19.82	"
Amylalcohol	$C_4H_9(CH_2)(OH)$	0.8296	88.0	14.48	14.62	"
					15.00	Cohn & Arons
Allylalcohol	$C_3H_5.OH$	0.85	58.0	21.54	21.60	Thwing
Cetylalcohol	$C_{16}H_{33}.OH$	0.818	242.0	6.56	6.42	"
Tartaric acid	$C_2H_2(OH)_2(CO_2H)_2$	1.75	150.0	35.02	35.90	"
Lactic acid	$CH.OH.CH_3.CO_2H$	1.215	90.0	20.97	20.90	"
Creosol	$C_6H_3.OH.OCH_3.CH_3$	1.0894	138.0	12.63	11.75	"
Formic acid	$OH.COH$	1.223	46.0	61.88	62.00	"
Aldehyde	$CH_3.COH$	0.807	44.0	18.55	18.55	"
Salicylaldehyde	$C_6H_4.OH.COH$	1.172	132.0	20.46	19.21	"
Propylaldehyde	$C_2H_5.COH$	0.866	58.0	14.52	14.41	"
Benzaldehyde	$C_6H_5.COH$	1.063	106.0	11.70	14.48	"
Valeraldehyde	$CH_3(CH_2)_3COH$	0.884	86.0	11.92	11.76	"
Cuminaldehyde	$C_6H_4(C_3H_7)COH$	0.9832	148.0	8.37	10.68	"
Chloral	$CCl_3.COH$	1.549	147.5	13.43	5.47	"
Acetone	$CH_3.CO.CH_3$	0.792	58.0	21.85	21.85	"
Methylethylketone	$CH_3CO.C_2H_5$	0.8125	72.0	18.42	18.44	"
Acetylchloride	$CH_3.CO.Cl$	1.138	78.5	24.06	25.30	"
Methylpropylketone	$CH_3.CO.C_3H_7$	0.828	86.0	16.06	16.75	"

TABLE XIII. (*continued*).

Substance.	Formula.	D	M	K Computed.	K Observed.	Observer.
Acetophenone	$CH_3.CO.C_6H_5$	1.032	120.0	15.08	16.24	Thwing
Dipropylketone	$C_3H_7.CO.C_3H_7$	0.82	114.0	12.52	12.44	"
Methylhexylketone	$CH_3.CO.C_6H_3$	0.818	128.0	11.41	10.42	"
Nitrobenzole	$C_6H_5NO_2$	1.20	123.0	32.19	32.19	"
Nitromethane	$CH_3.NO_2$	1.096	61.0	56.35	56.36	"
Nitrotoluole	$C_6H_4.CH_3.NO_2$	1.136	137.0	27.38	26.58	"
Ethylnitrate	$C_2H_5.O.NO_2$	1.132	91.0	35.30	17.72	"
Valeric acid	$CH_3(CH_2)_3CO_2H$	0.9577	102.0	3.15	3.06	"
Butyric acid	$CH_3(CH_2)_2CO_2H$	0.9587	88.0	3.20	3.16	"
					3.00	Tereschin
Toluole	$C_6H_5.CH_3$	0.886	92.0	2.37	2.37	Thwing
					2.36	Palaz
					2.30	Thomasewski
					2.37	Landolt & Jahn
Metaxylole	$C_6H_4(CH_3)_2$	0.878	106.0	2.41	2.38	Negreano
Paraxylole	$C_6H_4(CH_3)_2$	0.862	106.0	2.37	2.38	Thomasewski
					2.23	Landolt & Jahn
Carbon disulphide	CS_2	1.292	76.0	2.47	2.50	Thwing
					2.61	Palaz
					2.60	Quincke
					2.30	Weber
Sulphur	S	1.98	32.0	2.58	2.69	Mean: 12 observers
Gypsum	$CaSO_4$	2.30	232.0	5.60	5.61	Thwing
Copper sulphate	$CuSO_4$	2.274	249.5	5.51	5.46	"
Barium sulphate	$BaSO_4$	4.47	233.0	11.20	11.40	"
Lead sulphate	$PbSO_4$	6.38	302.0	15.72	15.80	"
" sulphide	PbS	7.30	238.4	17.91	17.92	"
Potassium sulphate	K_2SO_4	2.65	174.0	6.25	6.45	"

In the case of solids and of such liquids as contain no atom groups the atomic capacities seem to be in the ratio of the atomic weights, so that the formula takes the simple form $K=\kappa D$, where $\kappa=K_H$.

The values given in Table XIV. are reckoned by the formula $K=2.6D$. Sulphur requires the value $K_s=M\kappa/2$, instead of $M\kappa$, and is therefore entered with its compounds in Table XIII.

So close an agreement in the case of such a large number of substances can hardly be the result of chance.

TABLE XIV.

$K = 2.6\,D$

Substance.	Formula.	D	K		Observer.
			Computed.	Observed.	
Benzole	C_6H_6	0.898	2.32	2.31	Mean: 9 observers
Octane	C_8H_{18}	0.718	1.87	1.93	Landolt & Jahn
Decane	$C_{10}H_{22}$	0.746	1.94	1.97	" " "
Petroleum	—	0.8	2.08	2.06	Mean: 10 observers
Turpentine	$C_{10}H_{16}$	0.856	2.23	2.23	" 12 observers
		0.87	2.26		
Citron oil	—	0.853	2.22	2.25	Thomasewski
Vaseline oil	—	0.863	2.24	2.17	Hopkinson
Chloroform	$CHCl_3$	1.526	3.97	3.95	Thwing
Bromoform	$CHBr_3$	2.83	7.36	7.42	"
Paraffine	—	0.87 to 0.91	2.26 to 2.36	2.18	"
				2.32	Boltzmann
				1.99	Gordon
				2.28	Hopkinson
				2.17	Winkelmann
				2.31	Donle
Ebonite	—	0.97	2.52	2.53	Mean: 12 observers
Porcelain	—	1.739	4.52	4.20	Thwing
Glass (mirror)	—	2.4	6.24	5.84	"
Sandstone	—	2.36	6.18	6.20	"
Sand	—	2.8	7.28	7.43	"
Marble	$CaO.CO_2$	2.65	6.89	6.14	"
Iceland spar	$CaO.CO_3$	2.75	7.20	7.35	"
Rock salt	$NaCl$	2.16	5.61	5.81	"
Cane sugar	$C_{12}H_{22}O_{11}$	1.58	4.12	4.16	"
Ice	H_2O	0.92	2.50	2.85	"
Glacial acetic acid	$C_2H_4O_2$	1.06	2.76	2.79	"
Potassium chlorate	$KClO_3$	2.33	6.06	6.18	"
" carbonate	K_2CO_3	2.29	5.95	5.62	"
Copper oxide	CuO	2.65	6.25	6.45	"
Iron oxide	FeO	5.12	13.31	14.20	"
Barium nitrate	$Ba(NO_3)_2$	3.23	8.39	9.15	"
Lead carbonate	$PbCO_3$	6.57	17.08	18.58	"
" oxide	PbO	9.29	25.15	25.90	"

Water, acetic acid, and sugar behave in the solid form exactly as if the atom groups to which they seem to owe their high specific capacity in the liquid form were as such no longer present.

If we assume this to be the case, the remarkable variation of water with the temperature as well as the anomalous behavior of alcoholic mixtures could be explained on the supposition that a complete association of the elements to hydroxyl occurs only at a definite temperature and under conditions of absolute purity. The thought at once suggests itself that where the singular points occur in the mixture curves, a complete chemical union takes place, and we are dealing, not with a mixture, but with a pure substance.

If we bear in mind that hydroxyl is nearly one and one-half times as dense as water, and does not freeze at a temperature of $-30°$, our hypothesis will explain the density maximum of water at $4°$, and the fact that water may be cooled to a point several degrees below the freezing-point without its freezing, the effect of the jar, which finally causes the water to freeze, being to set in motion the dissociation of the hydroxyl.

The complete establishment of the hypothesis here put forward requires, of course, a larger volume of experimental facts than are yet at hand. The extremely simple method, however, — which I trust will be made still more exact by others, if not in further experiments of my own, — offers a ready means of obtaining the necessary facts on which to base a theory of dielectrics. The method is but one of many applications which might be made of the beautiful discovery of Hertz to the solution of electrical problems.

MINOR CONTRIBUTIONS.

The Physical Laboratory of Adelbert College.

By Frank P. Whitman.

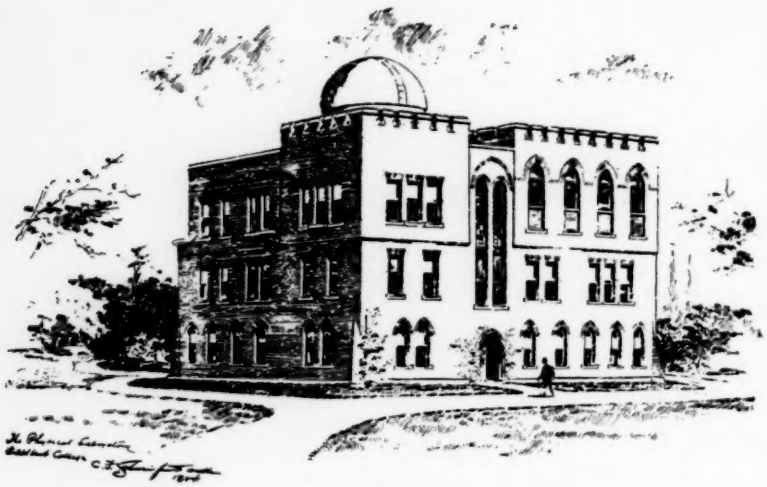
THE Physical Laboratory of Adelbert College, now in process of construction, is a building measuring about eighty feet by sixty, and three stories in height. It has no basement, the floor of the first story being just above the level of the ground. It is built of hard-burned brick set in black mortar. The construction is of the so-called "mill" or slow-burning type, but with unusually heavy timbers and floors. The building is to be heated and ventilated by a blower, forcing air through brick conduits. The steam to drive the blower, heat the air, and supply power, will be taken from another building. By this method of heating, and the construction, the presence of iron in the building is almost completely avoided.

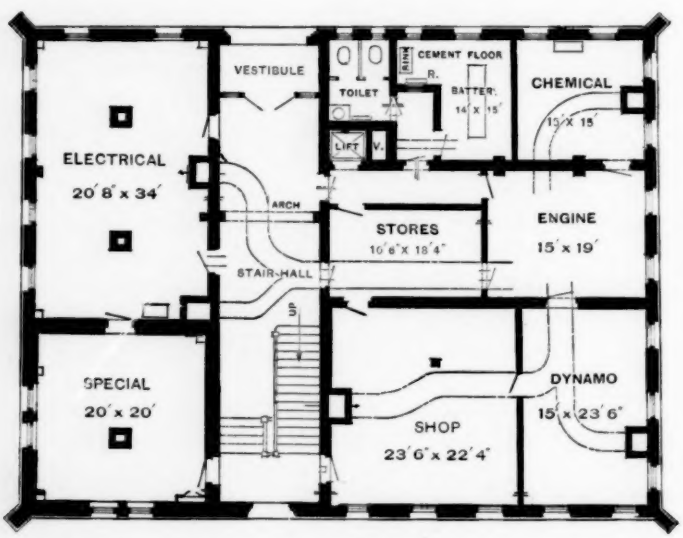
The stairway hall divides the building into two parts, the one on the north containing the large rooms necessary for the elementary classes, the southern portion divided into smaller parts for more advanced work. In this part of the building a tower twenty feet square with heavy walls adds to the general stability of the structure, and is arranged to support, if required, a telescope and dome. It is not, however, certain that this feature will be added at present.

The Main Laboratory, on the second floor, is made long and narrow to secure good lighting. It will accommodate forty students or more. Tables fastened on wall-brackets will run around three sides of the room. A large stone slab (marked "Table" on the plan), flush with the floor, resting on a pier, will afford additional support for apparatus requiring steadiness.

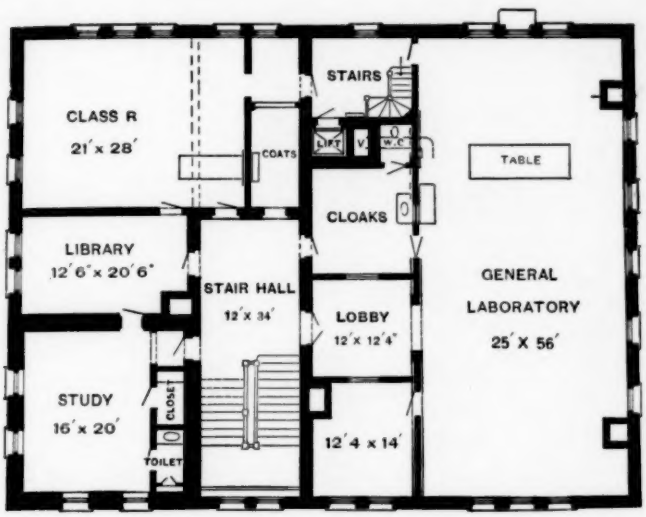
A small stairway ascends from this room to the cabinet of lecture apparatus above, making it easy to transfer instruments when required from floor to floor. Most of the elementary apparatus will be stored in the Laboratory and the small rooms adjoining.

The Lecture Room is seventeen feet high, and will accommodate about two hundred. It is lighted by four windows and a large skylight, which will be darkened, when necessary, by light opaque screens, running in grooves

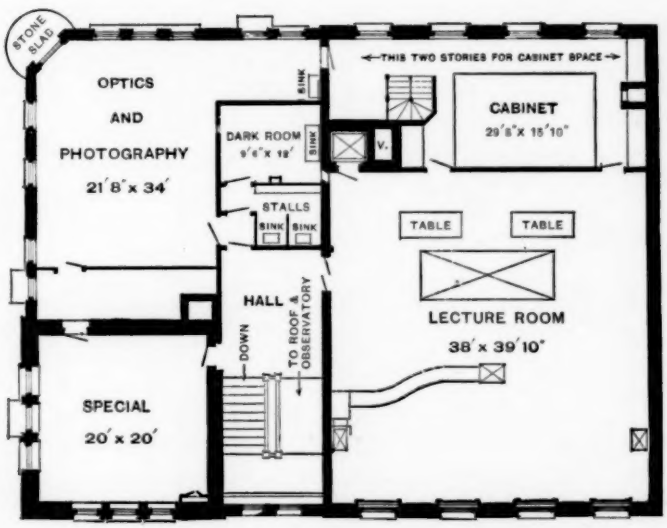




Ground Story.



Second Story.



Third Story.

like a window sash, and controlled by mechanism at the lecture table. The apparatus cabinet, just behind the Lecture Room, is divided into two stories, each eight feet high, connected by a short stairway. All shelves are thus brought into reach from the floor.

The corner looking directly south (the building faces southwest) is cut off in this story to make room for a large window with shelf for heliostat.

The Laboratory will be substantially constructed throughout, and well though plainly finished. The architect is C. F. Schweinfurth of Cleveland. The building, with necessary furnishings, is the gift of Mr. Samuel Mather.

A LABORATORY EXPERIMENT IN SIMPLE HARMONIC MOTION.

BY JOHN O. REED.

ALTHOUGH the interest in the study of simple harmonic motion is a constantly increasing one, yet simple and practical laboratory experiments in this subject are by no means common. The following is offered as likely to prove useful and suggestive: —

An elastic spiral, if suspended by one end and carrying a heavy ball attached to the other, will, if pulled down and released, execute approximately simple harmonic vibrations whose period may be accurately computed. The experiment consists in comparing the computed period of the oscillations of such a system with that determined by direct observation. If we neglect the weight of the spiral and assume Hooke's law to hold throughout the motion, we may derive our working formula as follows:[1] Letting b be the extension of the spring due to the weight W, c its extension at the beginning of the motion, x the extension at any time t, S the corresponding tension of the spring, then, by Hooke's law, we have

$$S = \frac{W}{b} x.$$

The differential equation of motion is

$$M\frac{d^2x}{dt^2} = W - S,$$

or

$$\frac{d^2x}{dt^2} + \frac{g}{b}(x - b) = 0.$$

The solution of this, after determining the constants, is

$$x = b + (c - b)\cos\sqrt{\frac{g}{b}}\,t.$$

[1] Williamson's Dynamics, p. 157.

This represents a simple harmonic motion, the period of which is clearly

$$T = 2\pi\sqrt{\frac{b}{g}}.$$

Hence for laboratory purposes we need to measure simply the extension b due to the weight W in order to compute the period of the oscillation, if we suppose the value of g to be known. For the determination of the period T by observation, the following method was adopted: A piece of $\frac{5}{8}$ inch gas-pipe, one meter long, has at its upper end a movable clamp a, Fig. 1, which supports the spiral spring in front of a mirror scale bc, etched in millimeters for reading off directly the length of the spring when free, and when stretched by the ball g. The difference between these readings gives the value b in the formula. The wire connecting the ball to the spring passes through a cylindrical mercury cup d by means of a small glass tube t, around which the mercury in d forms a ring about one centimeter in width and six centimeters deep. Electrical contact is made by means of the light steel wire s dipping into the mercury whenever the spring is extended beyond the stretched length due to the ball g. A tuning-fork, Fig. 2, a cylinder covered with smoked paper and provided with an electric scriber o, actuated by the current which is interrupted at the contact in d, complete the outfit. To use the apparatus, the spring is drawn down and tied by a fine cotton thread passed through the ring in the under side of the ball g and around the arm h, which must be so adjusted as to allow the wire to clear the sides of the glass tube through which it passes. When the ball has come to rest, the fork is set in vibration, the thread burned off, and the cylinder turned steadily while the ball oscillates. In practice it is found convenient to insert a key in the battery circuit by means of which the scriber may be allowed to record only such vibrations as are relatively free from the slight damping effect of the air and the little plunger s. The period of the tuning-fork, armed with its light

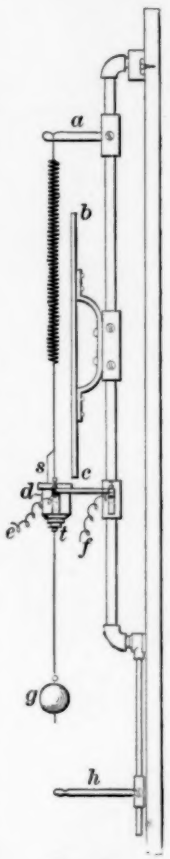


Fig. 1.

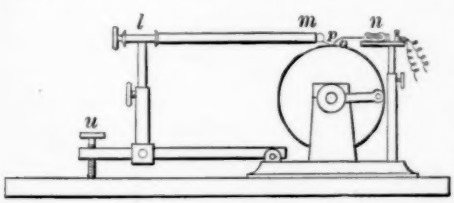


Fig. 2.

tracing-point, having been first carefully determined, the fork is thereafter used as the measurer of time, and the number n of waves between two successive movements of the scriber o, caused by the wire s plunging into the mercury, multiplied by the period of the fork, gives the period T of the spring in seconds.

As may be seen from the formula

$$T = 2\pi\sqrt{\frac{b}{g}},$$

an error of one millimeter in the reading of b would, if b were five centimeters, produce an error of 0.0045 of a second in T. Only gross carelessness could produce such an error as this. In fact, with nothing but the mirror scale the value of b may be readily determined to one-tenth of one millimeter, corresponding to an error of 0.00045 of a second. With cathetometer readings, the accuracy of this part of the experiment might be considerably increased. With a tuning-fork making 256 vibrations per second, it is easy to determine time by means of the smoked paper to at least 0.0005 of a second, although very satisfactory results may be obtained from the use of a fork of twice this period.

Some idea of the accuracy of the method may be formed from the following results taken from students' laboratory note-books: —

SPRING NO. 1. — STEEL WIRE.

Period of fork $\frac{1}{65.5}$ sec.

$b = 6.81$ cm. $n = 34.26$.

T.

Computed, 0.5238 second.
Observed, 0.5230 "
Difference, 0.0008 "

SPRING NO. 2. — BRASS WIRE.

$b = 6.12$ cm. $n = 32.5$.

T.

Computed, 0.4964 second.
Observed, 0.4962 "
Difference, 0.0002 "

SPRING NO. 3. — BRASS WIRE.

$b = 5.02$ cm. $n = 29.53$.

T.

Computed, 0.4492 second.
Observed, 0.4508 "
Difference, 0.0016 "

The apparatus may also be used for the determination of g, although in this case the effect of slight errors of observation is obviously much greater than before. By furnishing the wire with a light tracing-point instead of the small wire s, and mounting the cylinder vertically, the same apparatus may be used to produce excellent specimens of damping curves, from which the retardation due to friction may be studied.

PHYSICAL LABORATORY OF THE UNIVERSITY OF MICHIGAN.

SOME EXPERIMENTS IN ELECTRIC PHOTOGRAPHY.

BY FERNANDO SANFORD.

DURING the past three years, I have been occasionally engaged in some experiments upon electric photography which may be of interest to the readers of the *Review*, notwithstanding the fact that Professor F. J. Smith, in his recent announcement of his "Inductoscript," has published similar results to those which I have obtained.

In 1891 I developed a negative image upon a piece of bromide paper which had been placed between, and in contact with, two pieces of metal which were connected with the poles of two chromic acid cells. These negatives showed only the places of contact of the sensitized side of the paper and the one piece of metal. During the same year, I developed characters written upon a sensitized plate with the end of a wire, the other end of which was connected to one terminal of an induction coil. In September, 1892, I read the article upon Breath Figures, by Mr. W. B. Crofts, in the *Philosophical Magazine* of August, 1892, and it occurred to me at once that these figures might be developed upon a photographic plate. I at once tried the experiment of laying the coin upon the sensitized side of the plate, and connecting it with the terminal of a small induction coil, capable of giving a spark of three or four millimeters, while a piece of tin foil upon the opposite side of the plate was connected with the other terminal of the coil. From the very short notice which I have read of Professor Smith's Inductoscript, this is exactly the method which he is now using, except that he uses a higher potential and exposes the plate only a second, while I exposed the negatives which I made from half an hour to an hour.

Several negatives were made in this way, the accompanying photograph, No. 1, being from one of them. With one exception, they all show a fringe around them, due to the escape of the charge from the edge of the coin, which accounts for the formation of the dark ring observed around the breath figures made upon glass.

Last September I undertook to photograph in the same way objects insulated from the photographic plate, and have since made negatives of coins separated from the plate by paraffine, shellac, mica, and gutta percha. The accompanying photograph, No. 2, was made on Oct. 7, the coin being insulated from the photographic plate by a sheet of mica about 0.04 mm. thick. The mica was laid directly upon the film side of the plate, and the coin was placed upon it and connected to one terminal of the small induction coil already mentioned. A circular piece of tin foil of the circumference of the coin was placed upon the glass side of the plate directly opposite the coin, and was connected to the other terminal of the induction coil. The little condenser thus made was clamped between two boards, and was covered up in a dark room. Two small discharging knobs were also attached to the terminals of the induction coil, and were separated by a space of less than a millimeter, so that when a single cell was connected with the primary coil, the spark between the knobs seemed continuous.

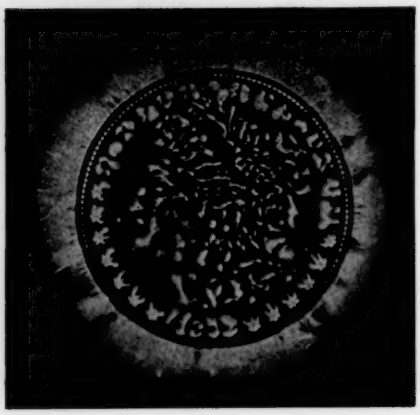


Fig. 1.

The plate was exposed to the action of the waves set up in this condenser for one hour, when it was taken out and the negative image developed upon it by the usual process. This photograph was made before I had heard of Professor Smith's process.

Fig. 2.

Several interesting facts are shown by these photographs, and it has seemed to me that the process might furnish a valuable method of studying some of the phenomena of condensers. The fact that the electric waves sent off by the condenser plates are propagated in a direction perpendicular to the plane of emergence is plainly shown by some of the negatives in my possession. The tendency for the charge to escape from the surfaces of greatest curvature is plainly shown, as in photograph No. 1. The surface of a coin which had been stamped with

fine parallel lines gave no image upon the plate, on account of the dispersion of the waves from its surface. In the case of a medal with a head of Washington in high relief, the whole head was left blank, but was outlined by a fringe like that around the edge of photograph No. 1.

I did not succeed in making a good negative through a thickness of insulator as great as one millimeter, but blackened spots of the size of the coin were made through several millimeters of gutta percha, even with the low potential mentioned above. Unquestionably, with a greater sparking distance, the effect could be observed throughout a condenser of much greater thickness.

STANFORD UNIVERSITY, CAL.,
Jan. 6, 1893.

THE ELECTRICAL CONDUCTIVITY OF COPPER AS AFFECTED BY THE SURROUNDING MEDIUM.

(1) *A Reply to Professor Carhart.*

BY FERNANDO SANFORD.

IN the March-April number of the *Physical Review*, Professor Henry S. Carhart publishes the results of an investigation which he says was carried on under his supervision for the purpose of confirming or refuting some conclusions reached by me in regard to the dependence of the electrical conductivity of a copper wire upon the nature of the surrounding medium.[1] In this investigation reported by Professor Carhart, the conductivity of a copper wire was measured in air, in alcohol, and in kerosene, and no difference in its conductivity was observed; hence he regards my conclusions as refuted, especially as he claims for his measurements a much higher degree of accuracy than was possible with the apparatus which I used.

Without discussing the question whether a set of measurements made with a much finer wire than was used by me and in two dielectrics besides air would furnish a sufficient basis for denying the accuracy of my observations made, as my published papers will show, with much greater care and with fifteen different dielectrics besides air, I wish in this article merely to discuss Professor Carhart's claims to a more accurate method of observation, and to call attention to some serious discrepancies between his published data and the graphical representation of them which he has given in his article.

[1] Leland Stanford Junior University Publications, Studies in Electricity, No. 1; Phil. Mag., January, 1893, Vol. 35, No. 212, p. 65.

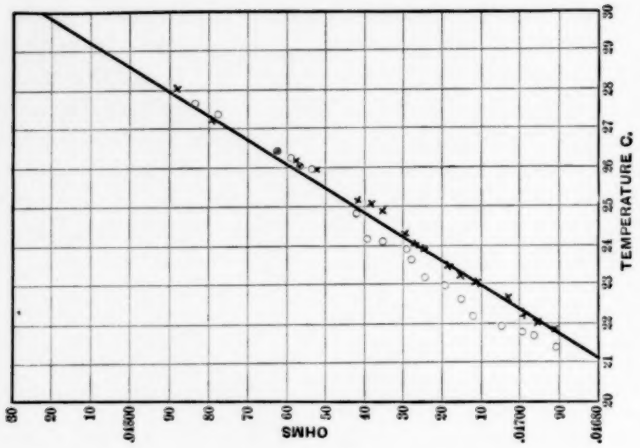


Fig. 1.

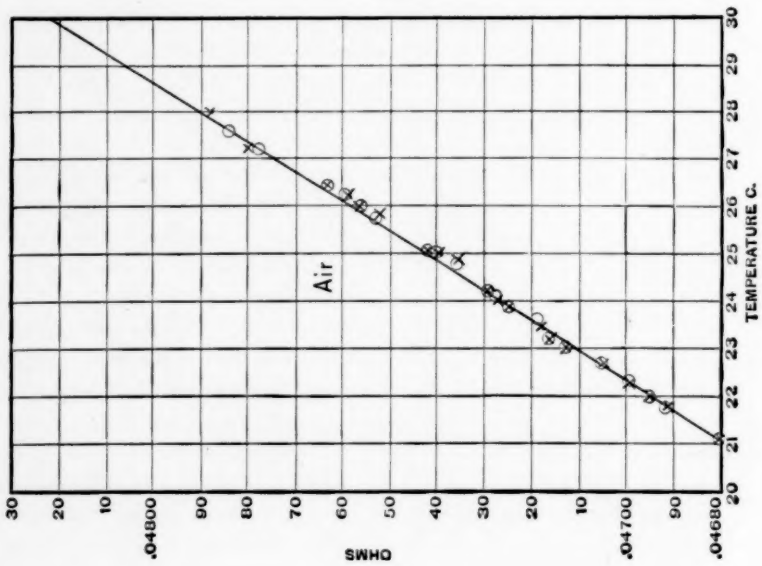


Fig. 2.

The general arrangement of the resistance to be measured was the same in Professor Carhart's investigation as in mine. A copper wire was stretched lengthwise through the middle of a copper tube, making electric contact with one end of the tube and passing out through an insulating stopper or plug in the other end, so that the current could be sent one way through the tube and return through the wire. The tube used by Professor Carhart was 86.3 cm. long. The tube used by me was 120 cm. long. The diameter of the wire used by Professor Carhart was 0.7 mm. The wire used by me in the experiments referred to by Professor Carhart was 1 mm. in diameter. The resistance of Professor Carhart's tube and wire at 21 c. was 0.0468 ohm, while the resistance of the tube and wire used by me was, at the same temperature, 0.338 ohm. It will thus be seen that the tube and wire used by Professor Carhart had a resistance of 140 per cent, and a surface (considering the wire alone) of only 50 per cent the tube and wire used by me.

The temperature was in both cases measured by a thermometer inserted into a side tube with its bulb in contact with the wire. In Professor Carhart's measurements, the thermometer was graduated to half degrees, while in my experiments a standard thermometer graduated to tenths was used. A change of temperature of one-tenth of a degree made, in Professor Carhart's wire, a change in resistance of 0.000016 ohm. The same temperature change made a change in resistance of 0.000012 ohm in the wire used by me.

The galvanometer described by me in the article referred to by Professor Carhart would not give a noticeable deflection upon reversing the current for a change of resistance of less than 0.00002 ohm, but by observing the deflections in opposite directions upon changing the resistance by 0.0001 ohm, the resistance was estimated to 0.00001 ohm. While a high degree of accuracy was not claimed for this method of estimating the value of the figure in the fifth decimal place, it would not compare unfavorably with the accuracy of estimating temperature to tenths of a degree with a thermometer graduated to half degrees. The complete notes published with my first article show that a variation of two in the fifth decimal place from the straight line representing the temperature curve of the wire was very rare, and that in no case could there be an uncertainty of one in the fifth decimal place as to the true position of the curve.

Professor Carhart seems to have made a special effort to secure accurate resistance measurements, and claims for his galvanometer and bridge combination a sensitiveness of at least two and one-half times as great as mine. To this part of his claim I have no objection to offer. Whether the resistance could be accurately measured to the fifth or the seventh decimal place is of no consequence. The fact remains that the smallest temperature variation measured by Professor Carhart's thermometer would corre-

spond to a difference of resistance of 0.00008 ohm, and granting that he could estimate the temperature with a fair degree of accuracy to one-tenth of a degree, this would correspond to a difference of resistance of 0.000016 ohm. I accordingly fail to see wherein the apparatus employed by Professor Carhart had an advantage in sensitiveness over that employed by me. On the other hand, if the methods of measurement employed in the two investigations are considered, it will be seen that my method has the advantage of much greater accuracy than that described by Professor Carhart. Manifestly, the most uncertain factor in the whole measurement is the temperature of the wire. This is especially true when the tube contains air or another gas. What the thermometer really indicates is its own temperature, taken from the air very near the wire. If the temperature of the air is rapidly changing, this certainly is not the temperature of the wire. The thin wire would certainly take the temperature of the air more quickly than the mercury in the thermometer bulb, protected, as it is, by its non-conducting covering of glass. To guard against this possible difference in the temperature of the wire and the thermometer, my measurements were all made at the room temperature, and in a room where the total temperature change for twenty-four hours rarely amounted to 10°, and frequently did not reach 5°. They were also made at all hours of the day, and frequently late in the evening, with a period of several days (in one case a month) devoted to each set of measurements, so that approximately the same number would be made with a rising as with a falling temperature.

Professor Carhart's measurements were made by heating the tube up to about 30° with warm water, and then making a series of measurements as it cooled off. The published data would indicate that all the measurements were made with a falling temperature. With the tube filled with a liquid and cooled slowly enough, this method might be made to give sufficiently accurate temperature measurements, but with air in the tube, a much greater time of cooling would be necessary to give the same accuracy. Professor Carhart's published data show that he has not overcome this difficulty. The data given in Tables I. and II., page 326, are all the data he has seen fit to publish regarding his resistance measurements in air, and the two curves platted from them show no close relation to each other, either in direction or position, notwithstanding the fact that Professor Carhart says in referring to them, page 325, "There is no difference between them."

In order to show the high degree of accuracy of Professor Carhart's method somewhat better than it is shown in the curve given by him, I have platted accurately upon millimeter paper the points representing the resistances and the corresponding temperatures, as given in Tables I. and II.,

and have drawn through them a line representing the position of Professor Carhart's curve, Fig. 2, page 329. I have then copied these points upon tracing cloth laid over the ruled paper, and have photographed the figure thus given, reducing it to the scale of Professor Carhart's published diagram. The figure thus photographed is shown as Fig. 1 of the present article, while Fig. 2 is a reproduction of the diagram published by Professor Carhart as representing the same data. The circles, as in Professor Carhart's diagram, show the points taken from Table I., and the crosses the points taken from Table II. It will be seen that in Professor Carhart's diagram there are thirteen pairs of coinciding points out of the forty points given, while in reality there are but two such coinciding pairs given in the two tables. Beginning at the lowest temperature, the first seven pairs of points are made to coincide in Professor Carhart's diagram. Most of these points thus brought into coincidence were made at temperatures of about half a degree apart. The points given in Table I. frequently fall 0.00007 or 0.00008 ohm above the curve drawn, and one point, made at 22°.3, falls 0.00014 ohm above the curve.

And this is the air curve which Professor Carhart has found to coincide with all his other curves, thus refuting my conclusions.

STANFORD UNIVERSITY, CAL.
March 31, 1894.

(2) *Reply to Professor Sanford.*

BY HENRY S. CARHART.

The only part of Professor Sanford's criticism to which I wish to make particular reply, is that in which he plats the data of my Table I., and creates the impression that I have warped the data to suit my statements. The fact remains that Fig. 2 of my paper, reproduced by Professor Sanford, is entirely correct, or as nearly so as one should expect in so small a diagram. The error is in the table. The diagrams were plotted from the original data on engraved millimeter paper, and were transferred from that to the drawings for the engraver. But, I regret to say, the table has an error in it, made by Mr. Keeler in copying.

If Professor Sanford had run down the column from the top, he would have found the points right for the first six temperatures. Then the discrepancy begins. But if he had examined the diagram carefully, he would have found platted two interpolated temperatures not found in the table. The others then follow in order, and not half a degree, as he asserts, out of the way. The column of temperatures was copied with the omission of two values where the discrepancy begins, so that the resistances are two

points out of step with the temperatures. Fortunately, Mr. Keeler has the original data. I reproduce the resistances and temperatures of Table I., together with the correct temperatures from which the small circles of Fig. 2 were platted. From this it will be seen that a blunder was made in copying the table, but the diagram is correct, and there was no warping of facts to suit any one's purpose.

R	*t*	Correct *t*
0.04784	27°.6	27°.6
78	.3	.3
63	26°.4	26°.4
59	.25	.25
57	.0	.0
54	25°.9	25°.9
42	21°.8	[25°.1]
40	.2	[25°.0]
36	.1	24°.8
29	23°.9	.2
28	.65	.1
25	.2	23°.9
19	.0	.65
16	22°.7	.2
13	.3	.0
05	.0	22°.7
0.04699	21°.8	.3
96	.6	.0
91	.3	21°.8
80	.1	.2

The numbers omitted I have enclosed in brackets. Hence Fig. 2 was correctly platted, and it remains the correct air curve, which I have found to coincide with all the other curves. If they refute Professor Sanford's conclusions, it is the refutation of facts.

As to the use of a thermometer reading to half degrees instead of tenths, the choice was made deliberately. If an error of a tenth, or even a half, degree were made on individual readings, the only effect would be to make the points scatter more on either side of the best line representing them; for on an average, as many readings would be too high as too low. But notwithstanding Professor Sanford's use of a tenth-degree thermometer, our points lie more nearly on a straight line than do his.

The observations were all made on a falling temperature, but the tube was heated by a large mass of water in a trough immediately beside it, and was covered over with waste or other poor conductor to prevent loss of

heat by radiation and convection. It is not at all probable that the thermometer lagged appreciably behind the wire with a temperature falling only three or four degrees an hour; and if it did, it would be strange indeed if in every case this lag should be just sufficient to eliminate the difference between the air and alcohol or other curves, which Professor Sanford ascribes to the influence of the dielectric.

(3) *Final Reply to Professor Carhart.*

By Fernando Sanford.

I have read Professor Carhart's explanation of the discrepancy between his diagram and his published data. It is entirely satisfactory. If I created the impression in Professor Carhart's mind that he had warped his data, I did it by simply stating the fact that his data and the diagram did not correspond. I offered no explanation for the discrepancy. With the corrected data which Professor Carhart gives, the printed diagram corresponds closely enough for the purpose. If, as it now appears from the data, the two tables were made at the same time by merely reversing the current and taking two readings at the same temperature, I was wrong in taking one curve as a measure of the accuracy of the other. The coincidence of the points merely shows that the same galvanometer deflection and temperature were read twice alike.

The question of the refutation of my observations is not under consideration. How Professor Carhart can suppose that an observation which his students did not make can refute my statements about an observation which I did make, I am unable to understand. Granting that the wire used by Professor Carhart's students showed no difference in resistance in the dielectrics in which it was tested, it refutes nothing which I have said. Since the publication of my original paper, I have accumulated a much larger amount of data upon the same subject which I shall publish later. My only reason for replying to Professor Carhart's article was his assumption of superior accuracy in his work. I think I have shown that notwithstanding his special effort at refinement in resistance measurement and his deliberate choice of a thermometer graduated to half degrees, his apparatus was not capable of giving as accurate results as was the apparatus which I used, and his precautions against error were not nearly so great as were mine.

NOTE.

August Kundt. — The news of the death of Professor Kundt, Director of the Physical Laboratory in Berlin, who passed from this life at his country place in Israelsdorf, near Luebeck, on the 21st of May, 1894, comes to very many physicists in America as a message of personal bereavement. During his sixteen years as director of the laboratory in Strasburg, and during the eight years following upon that when he was at the head of Physical Institute of the University of Berlin, Kundt numbered among his pupils many Americans upon all of whom, it is safe to say, he made an impression never to be effaced.

August Adolf Eduard Eberhard Kundt was born on the 18th of November, 1839. In 1860 he was matriculated at the University of Leipsig as a student of mathematics and the sciences. In his third semester he entered at Berlin, where he completed his university career. In Berlin he became the pupil of Magnus, who gave him a place in his private laboratory. Kundt received the degree of Doctor of Philosophy at Berlin in 1864, the title of his thesis being *De Lumine Depolarisato*. He began his career as a gymnasial teacher in that city, and in 1867 he registered himself as privatdocent in the University. In the following year he was called to the Polytechnic Institute of Zürich as Professor of Physics, and in 1870 he became the successor of Clausius in the University of Wuerzburg. Upon the reestablishment of the University of Strasburg, at the close of the Franco-German war, he was made director of the Physical Laboratory, which position he held until 1888. In that year Von Helmholtz resigned the directorship of the Physical Laboratory in Berlin, and Kundt, under whose administration the Department of Physics in Strasburg had reached a position second to none in the empire, was called to be his successor. This position he held till the end of his life. To the further development of physics in Berlin, he gave the efforts of his ripest years. To it, indeed, it may be said, he sacrificed his life, for there can be no doubt that the increasing burden thus laid upon him hastened his death.

Kundt was above all an experimental physicist. He is best known to the scientific world through his investigations upon the velocity of sound upon the specific heat of mercury, a research which he carried out in collaboration with Warburg upon the polarization of light and upon the index of refraction of metals. The first of these is perhaps his most widely

known work, his ingenious method of measuring wave-lengths by means of dust figures having become classical. It is noteworthy that this research was completed while he was still an undergraduate in the University of Berlin. Among those who were fortunate enough to be his pupils, Kundt will be remembered not only for his fertility in experimental research, but also for his contagious enthusiasm for all that related to physics, and his extraordinary skill as a demonstrator. One who had the privilege to sit under him during the last course of lectures which he gave says: "There was something inspiring and magnetic about his lectures which I have never noticed to the same extent with any one else. He seemed to address each hearer personally. In more direct personal intercourse, his power of arousing enthusiasm was still more marked. Best of all, he had a way of making one feel that his interest was a personal one and that he was a friend." These were the qualities which enabled Kundt to take up the directorship of the great laboratory in Berlin, which his illustrious predecessor, von Helmholtz, had laid down, extending its influence and stimulating the inmates to a pitch of performance unparalleled in the history of such institutions.

E. L. N.

NEW BOOKS.

Neudrucke von Schriften und Karten ueber Meteorologie und Erdmagnetismus. Von Prof. Dr. G. HELLMANN.

No. 2. *Blaise Pascal. Récit de la Grande Expérience de l'Équilibre des Liqueurs. Paris*, 1648. Facsimiledruck mit einer Einleitung. 4to. pp. 10, 20. Price, 3 marks (75 c.). Berlin, A. Ascher & Co., 1893.

In the volume before us, which forms No. 2 of the excellent and beautifully executed series of reprints in facsimile of epoch-making books and charts edited by that eminent meteorologist and bibliographer, Professor Hellmann, we have a book that will surely interest every physicist. It is the reprint of a work of the greatest rarity, as but three copies — two in Paris and one in Breslau (from which the reproduction was made) — could be found. It contains Pascal's *first* announcement to the world of the successful outcome of the all-important experiments carried out by his brother-in-law, Perier, at Pascal's suggestion. These experiments, as is well known, were the first to give experimental proof of the existence of atmospheric pressure, and hence of the real cause for the rise of mercury in an inverted tube exhausted of air — the memorable discovery of Torricelli (the experiment was actually performed by one of his pupils) in 1643. This great discovery completely overthrew the doctrine of nature's "*horror vacui*," which had been prevailing up to that time, and in which Pascal himself had believed (as appears from the *Récit*).

The first eight pages of the facsimile reprint form the letter of instructions sent to Perier by Pascal, November, 1647. On September 22, 1648, Pascal received the glad tidings of the successful outcome of the experiments. This letter of Perier is given in pp. 17–20. As no small share of the credit for the discovery must be given to Perier, on account of the scrupulous care with which he set about his task, it may not be amiss to briefly outline the method employed. Perier filled two tubes with mercury and found that they read the same at Clermont. Emptying one of them, he ascended with it the Puy de Dome (3550 feet above Clermont) and again took a barometric reading. It was found less than before. After repeating the experiment at a station half-way down, and arriving at Clermont, the two tubes were again read and found in perfect accord. The true explanation of the Torricellian experiment thus became

at once evident. So careful was Perier that he even surrounded himself with witnesses. Immediately upon receiving Perier's letter, Pascal hastily drew up a tract, concluding with the "consequences," which, as it appears, were added to after the distribution of the tract, and the additions printed on a separate slip, which was found pasted in the Breslau copy. And it is this interesting "messenger" which has now been made accessible for the first time to every one. Professor Hellmann and his co-operators, the German Meteorological Society and its Berlin branch, have thus merited the warmest praise from every scientist.

Pascal's tract was almost bodily incorporated in the *Traitez de l'Équilibre des Liqueurs*, etc., published by Perier in 1663, one year after Pascal's death, and this work is generally accredited with the first account of the discovery.

As to the great question whether Pascal was the real originator of these experiments, or *Descartes*, who asserted in two letters of 1649 to Carcavi that *he* had suggested them to Pascal (which statements Pascal never denied), Professor Hellmann states in the introduction that, after carefully examining all the evidence, he has to agree with Mr. Mourisson, who in his recent book[1] has decided in favor of Descartes. Be that as it may, the fact remains that we owe to *Pascal* the first promulgation of the "great secret" to the world.

It may not be amiss to say a word or two with respect to this interesting series of reprints. The title itself indicates that they are to have special reference to Meteorology and Terrestrial Magnetism. As, however, both these subjects are, in the broadest sense, but Applied Physics to Terrestrial Phenomena (or should be), these facsimile reprints cannot fail to create a wider interest. Moreover, the physicist and the mathematician seem to be waking up to the fact that here are two fields, comparatively unexplored, that present some of the most fascinating problems, and offer some of the prettiest applications of physical laws and mathematical formulæ with which nature has ever found pleasure in taxing and baffling the human mind.

No. 1 of the series bears the following title: *L. Reynman. Wetterbuechlin. Von wahrer Erkenntniss des Wetters.* 1510. Facsimiledruck mit einer Einleitung. 4to. pp. 42, 14. Price, 6 marks ($1.50). Berlin, A. Ascher & Co. 1893.

This, as will be seen from the title, will be most interesting to the professional meteorologist, though the bibliographer, the student of folk-lore, and others may find it no less so. It is the oldest meteorological work in the German language, being first published in 1505 — of this edition all trace has been lost. It next appeared in 1510, and of this but one copy is extant, and in possession of Professor Hellmann. From it the elegant and attractive reproduction was made.

[1] Pascal, Physicien et Philosophe, etc. Paris, 1888.

So successful was Reynman's little work that it ran through seventeen editions in thirty-four years, of all of which scarcely more than three dozen copies can now be found. It was incorporated in a number of works, and almost literally translated in the *Boke of Knowledge of Thynges Vnknown, appertaining to Astronomy, with certain necessary Rules,* published in London, 1585. The chief reason for this remarkable "run" lay in the fact that the author's aim apparently was to base *his* weather proverbs (many in rhyme) and weather signs upon *natural* laws and *natural* phenomena. This differentiated it at once from the current literature of the day, in which astrology played so prominent a part. In the introduction to the reprint, Professor Hellmann, in his scholarly way, gives the book a careful and critical discussion.

These "Neudrucke," as the circular announces, will all appear in elegant and substantial form,—small quarto ($20\frac{1}{2} \times 25\frac{1}{2}$ cm.),—and each year two at the most will be issued. Each number can be bought separately. Among those promised are the following:—

Luke Howard, On the Modifications of Clouds, 1803. In this valuable work the first attempt at classification of cloud-forms is found. It is still a standard work. In the reprints the cloud-forms will be reproduced in facsimile. This number will shortly appear.

Edmund Halley's famous *Isogonic Chart, or Lines of Equal Magnetic Declination for the Epoch* 1700. As will be remembered, Halley is the originator of the method of representing the distribution of terrestrial phenomena by drawing lines through all those places on the earth's surface where the particular phenomenon (be it magnetic declination, temperature, etc.) has the same numerical value. His idea was taken up and applied to the magnetic inclination, it is usually believed, first by Wilcke in 1768. It does not appear to be known, however, and perhaps is pointed out here for the first time, that he was preceded by Whiston in 1721,[1] though, to be sure, the latter drew *his* isoclinic lines for but a very small portion of the earth; viz. Southern England and Northwestern France. Humboldt next took up the idea, and applied it to the intensity of terrestrial magnetic force, and also in 1817 to the distribution of temperature on the earth's surface, and thus gave birth to the *isotherms.* These latter it is also intended to reproduce in facsimile.

It is to be hoped that the German Meteorological Society, and its happily chosen editor, will meet with sufficient encouragement to be induced to continue in the good work.

L. A. BAUER.

[1] Will Whiston: Longitude and Latitude found by the Inclinatory or Dipping Needle, to which is appended Robert Norman's New Attractive, London, 1721. The said diagram will be found opposite p. xxvii. A copy of this work was found in the Royal Library of Berlin.

Vorlesungen über mathematische Physik. Vol. IV. Theorie der Wärme. By GUSTAV KIRCHHOFF. Edited by MAX PLANCK. Large 8vo, pp. x, 210. Leipzig, B. G. Teubner, 1894.

The fourth and last volume of this series is based upon lecture notes prepared by the author between the years 1876 and 1884, and has been edited, as was the third volume, by Kirchhoff's successor in the chair of Theoretical Physics at Berlin. Professor Planck has expressly refrained from introducing any alterations in treatment, except so far as such changes were found necessary in order to bring the notes into suitable form for publication. What few additions and explanations seemed desirable have therefore been placed in the form of foot-notes. The reader may thus feel assured that the method of presentation is that actually followed by Kirchhoff, and although it may at times appear that the editor has been over-cautious in keeping his own methods in the background, yet his work has been so carefully done that only in a few cases does the treatment appear really incomplete.

The general plan of the lectures is in accordance with a view expressed by Kirchhoff in the preface to his lectures on mechanics; namely, that theoretical physics should be looked upon as a *descriptive* science; that its object is to describe phenomena, and to express the observed relations between physical quantities as completely and as simply as is possible. The treatment throughout the book is consistent with this view. The phenomena first considered are those which require the fewest assumptions, and hypotheses as to the nature of heat and the molecular structure of matter are introduced only when they become necessary and useful. Especial attention is called to the fact that the fundamental laws of heat conduction and thermodynamics are independent of any kinetic hypothesis.

Since the study of conduction requires no assumption as to the nature of heat and the adoption of no absolute scale of temperatures, this branch of the subject is the one which is first discussed. The differential equations are at once deduced for the most general case, but the numerous examples which follow are in general simplified by the assumption that the bodies considered are both homogeneous and isotropic. Cases in which the conductivity is a function of the temperature, as well as cases in which the phenomena may be complicated by physical or chemical changes in the medium, are mentioned, but not discussed. The treatment in the four chapters devoted to conduction, while clear, exhibits no strikingly novel features, and possesses one fault that is very often met with in the discussion of partial differential equations, namely, that the reader is apt to gain no insight into the process of reasoning by which the author has reached his conclusions. While compelled to admit that the solutions are correct,

and that the methods used in obtaining them are ingenious, the student will have learned little at the end of these four chapters which will help him in solving similar equations for himself.

In the fifth chapter the subject of Thermodynamics is begun, and the author plunges at once into the consideration of Entropy and the Second Law. Carnot's cycle is first discussed, and by using the principle that heat cannot of itself pass from a colder to a hotter body, it is shown that the ratio of the heat taken from one reservoir to that given up to the other is a function of the two temperatures alone, and independent of the working fluid. The temperatures of the two reservoirs being θ_1 and θ_2, we have, therefore,

$$\frac{Q_1}{Q_2} = f(\theta_1\theta_2).$$

The next step consists in showing that

$$f(\theta_1\theta_2) = \frac{\phi(\theta_1)}{\phi(\theta_2)},$$

where ϕ is some function whose form is as yet unknown. $\phi(\theta_1)$ is now defined as the absolute temperature T_1, and the relation

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

follows at once for a simple reversible cycle. It will thus be seen that the thermodynamic definition of temperature is introduced at once, and is in fact the *only* definition given. It is not until several chapters later that a consideration of the behavior of the fixed gases enables the relation between the absolute and the ordinary temperature scales to be deduced. Although this procedure possesses disadvantages, it appears to me on the whole commendable.

Proceeding with the development of the Second Law, the author now shows that for *any* cycle the algebraic sum $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \cdots$ must be either equal to zero or negative. The statement of the Second Law is then put in the form

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \cdots \leqq 0,$$

without further comment. The lecture closes with an explanation of the term *entropy*, the proof that

$$\int \frac{\delta Q}{T} = 0$$

for a reversible cycle, and the final statement that the term *entropy* can with propriety be employed only when dealing with reversible processes.

I fear that this last statement will cause the thoughtful student some annoyance, since it appears to restrict the applications of the Second Law to quite narrow limits. In the following lecture these restrictions are partly removed by the statement that the term *entropy* may also be applied in the case of irreversible transformations, provided that the changes considered can also be brought about by reversible processes. The final step in the generalization is made by a foot-note of the editor, calling attention to the fact that no transformation has yet been observed which is *not* capable of being produced by such a process. The reader would have been saved some annoyance if these important statements had not been so widely separated.

The sixth chapter contains the application of the Second Law to isothermal changes, and to expansion in general. From Boyle's Law, and the fact that the specific heat for constant pressure is independent of the pressure, the laws of the perfect gas are developed as a special case. The three following chapters are devoted to applications of the law of Entropy to numerous purely physical problems, among which may be mentioned the dependence of melting and boiling points upon the pressure, Van der Waal's formula for the "imperfect" gases, critical points, and the absorption and development of heat in solution. It is to be regretted that the author has refrained from entering the domain of physical chemistry, so that applications of the Second Law to chemical transformations are wholly lacking.

Two chapters on the motion of fluids, illustrated by several interesting special cases, now follow, after which the discussion of the kinetic theory of gases is begun. The treatment of this subject, which occupies nearly half the book, is roughly in accordance with its historical development. Presented first in its simplest form, the theory is gradually developed by the introduction of new hypotheses, as these are shown to be necessary, and connected exposition of the subject is thus presented such as is rarely met with. Although by no means a complete treatise, this portion of the book presents so excellently the essential features of the theory that it will undoubtedly prove of great value.

In general, it may be said that if the volume before us is looked upon as a text-book, it will be found lacking in those concrete illustrations and incidental explanations which are so helpful to the student, but which rarely find their way into a lecturer's note-book. The logical and consistent method of treatment will, however, commend the book to all, and among those books which should be read in order that one may acquire a broad view of the theories of heat, Kirchhoff's lectures will take a prominent place.

ERNEST MERRITT.

The Mean Density of the Earth. By J. H. POYNTING, Sc.D., F.R.S. 8vo, pp. 176. London, C. Griffin & Co., 1894.

The author begins with a few statements regarding attractions, and shows by an example how the determination of the earth's mean density is connected with the law of gravitation. A valuable historical account follows, in which different methods are described, beginning with Bouguer in 1737, and ending with Laska in 1889. This is a very acceptable *résumé* for students of the subject, giving as it does the essential points of each method without becoming tiresome by going very much into details. Bouguer's formula on p. 11, for the diminution of gravity with elevation, seems to be erroneously written, and the error reappears on the following page. Bouguer's statement was that the decrease in gravity from the sea-level to the height h compared with the value at the sea-level is

$$\frac{2h\Delta - \frac{3}{2}h\delta}{r\Delta},$$

where r is the earth's radius, and δ and Δ are the densities of the intervening matter and the earth respectively. This reduces to

$$\frac{dg}{g} = \frac{2h}{r}\left(1 - \frac{3}{2}\frac{\delta}{\Delta}\right),$$

where g represents the value of gravity at the sea-level. The error is doubtless an oversight, but it should be pointed out for the benefit of readers who may wish to use the formula and who may not care to verify it, or may not have access to the original work. Of course the term $\frac{2h}{r}$ is the *decrease* on account of distance from the earth's center, and $\frac{3}{2}\frac{h}{r}\frac{\delta}{\Delta}$ is the *increase* on account of the matter lying between the sea-level and the upper station considered as a plain of infinite extent. Bouguer ultimately took $2\delta = \Delta$, from which the last term reduces to $\frac{3}{4}\frac{h}{r}$, and this combined with the first term gives $\frac{5}{4}\frac{h}{r}$ for the correction for both height and matter, which is the quantity now generally given as the total effect in the application of the so-called Young's rule. The quantity $\frac{3}{r}\frac{h}{\Delta}\frac{\delta}{\Delta}$, given at the top of page 12, is really the effect of a spherical shell, and was so used by Airy in the Harton Pit experiment.

The mountain work done in England, both by Maskelyne, and later in connection with the Ordnance Survey, is next noticed. The great ingenuity and elegance displayed in the application of Hutton's formula is one of the cardinal virtues of the mountain method, and it seems to us that

this should have been made more prominent by the author. The attraction of the mountain is given by a formula of triple integration where the successive integrations are in the direction of azimuth, horizontal distance, and height. Performing the operations indicated, the resultant force is

$$A = \delta h(\sin a_2 - \sin a_1) \text{ Nap. log } \frac{r_2}{r_1}$$

(Clarke's *Geodesy*, p. 295), a being the azimuth, and r the radii. From which it appears that if in making the compartments by means of azimuthal and radial lines, we take the sines of the azimuths in an arithmetical progression, and the radii of the circles in a geometrical one, the attractions vary as the heights, which simplifies enormously the computations. This is really the whole spirit of the method, and should be brought out prominently in any account of the work. Carlini's experiments on Mt. Cenis and Mendenhall's on Fujiyama falls under the same general head, — that of differences in the force of gravity, — and they depend on an estimation of the mountain density as given by specimens of the surface rock.

Another class of experiments is now described; viz. those where the pendulum is taken towards the center of the earth instead of being carried away from it.

Airy first conceived the idea of using a pendulum in a mine, and the experiment has been repeated in Austria by von Sterneck. The advantage of this is that a much more intimate knowledge of the material composing the intervening strata can be obtained than in the case of a mountain. The results, however, have not been very accordant. Indeed, a re-reduction of Airy's work by Haughton gives a value different from his by 20 per cent of the whole. Von Sterneck carried on the work at the surface and in the mine simultaneously, the time of oscillation of the two pendulums being determined by the same timepiece, the beats being transmitted telegraphically between the two stations.

The latter part of the historical account is devoted to what may be termed laboratory determinations. Instead of comparing the attraction of the earth with that of a mountain, or with that of a spherical shell of matter, as in the previous work, the attempt is now made to measure directly the attraction between known masses at a known distance apart.

Three different methods have been employed: Cavendish used a torsion balance, two lead balls being suspended by a copper wire, and deflected by large spheres of the same metal brought alternately to opposite sides of the attracted masses. This experiment was repeated by Reich and Baily. Cornu and Baille varied the method by using large globes of mercury for the attracting masses, and instead of shifting these masses bodily from one side to the other of the attracted ones, the mercury was pumped into

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an equal globe in the desired position. After the torsion balance comes an experiment by Jolly with the common balance, in which a mass was suspended at different distances, and its increase in weight noted for the lower position. Then a determination by Wilsing, where a pendulum balance was used, — a special feature of the work being that, by making the oscillations with varying masses, the value of the couple may be found without knowing the moment of inertia.

This completes the historical account. One or two experiments are referred to as being still in progress. A summary of results varying from 4.5 to 6.5 is given, from which it is seen that the mountain work gives the lowest values, the mine work the highest, and the laboratory experiments intermediate ones. Moreover, the laboratory results are more consistent than the others.

We now come to the principal part of the essay, being a determination by the author, following the common balance method. Two masses, each of about 22 kg. weight, are suspended from the arms of a finely constructed balance, and under each one of these is alternately brought a mass of about 150 kg. The quantity to be measured — the difference of weight — is doubled by this alternation. An ingenious method of eliminating the attraction of the large balls on the beam of the balance was employed by suspending the small balls at different distances, and equal pressures on all parts of the table before and after reversal were secured by placing on the table a mass equal to half that of the attracting mass, but at double its distance from the axis. This gave equilibrium at the same time that it did not very much alter the attractions. Some preliminary experiments, in which the attracted mass was 453 grams and the attracting mass 154,000 grams, gave unsatisfactory results on account of the small amount of the attraction (0.00001 gram), which led to the employment of larger balances and larger attracting masses. The attractions were then compared with the weights of riders. These were determined at the International Bureau of Weights and Measures at Paris by measuring their difference of weight taken two and two in all possible combinations, and then comparing the weight of all with a nearly equal known weight. The great accuracy of this method is at once apparent, as the separate weights are finally obtained through precise measure of small differences. The uncertainty of the results was not greater than one millionth of a gram. Poynting's work seems to have been done with great care, and is without doubt a valuable contribution to the literature of the subject. Space will not allow us to review the work in detail, — the essential and characteristic points being: the employment of the double suspension mirror method of detecting small motions; a graphic method of determining the center of swing; keeping the beam under a constant strain; and using nearly the same scale divisions for

deflections produced either by the attracting mass or by the riders. It was known that a movement of a centigram weight through a distance of 25 cm. on the beam would produce about the same deflection as that from the attracting masses, so that in the experiments one centigram weight was lifted and another let down at this distance to avoid moving them along the beam. The author so arranged his suspension mirror that the angle through which the balance beam turned was magnified 150 times, and the distance from the scale to the mirror was such that one division of the scale corresponded to an angular motion of 0.0001 radian. One scale division, therefore, implies an angular motion of about $\frac{1}{8}$ of a second of arc in the beam of the balance. By this method the motion of $\frac{1}{600000}$ of an inch in the masses is perceptible.

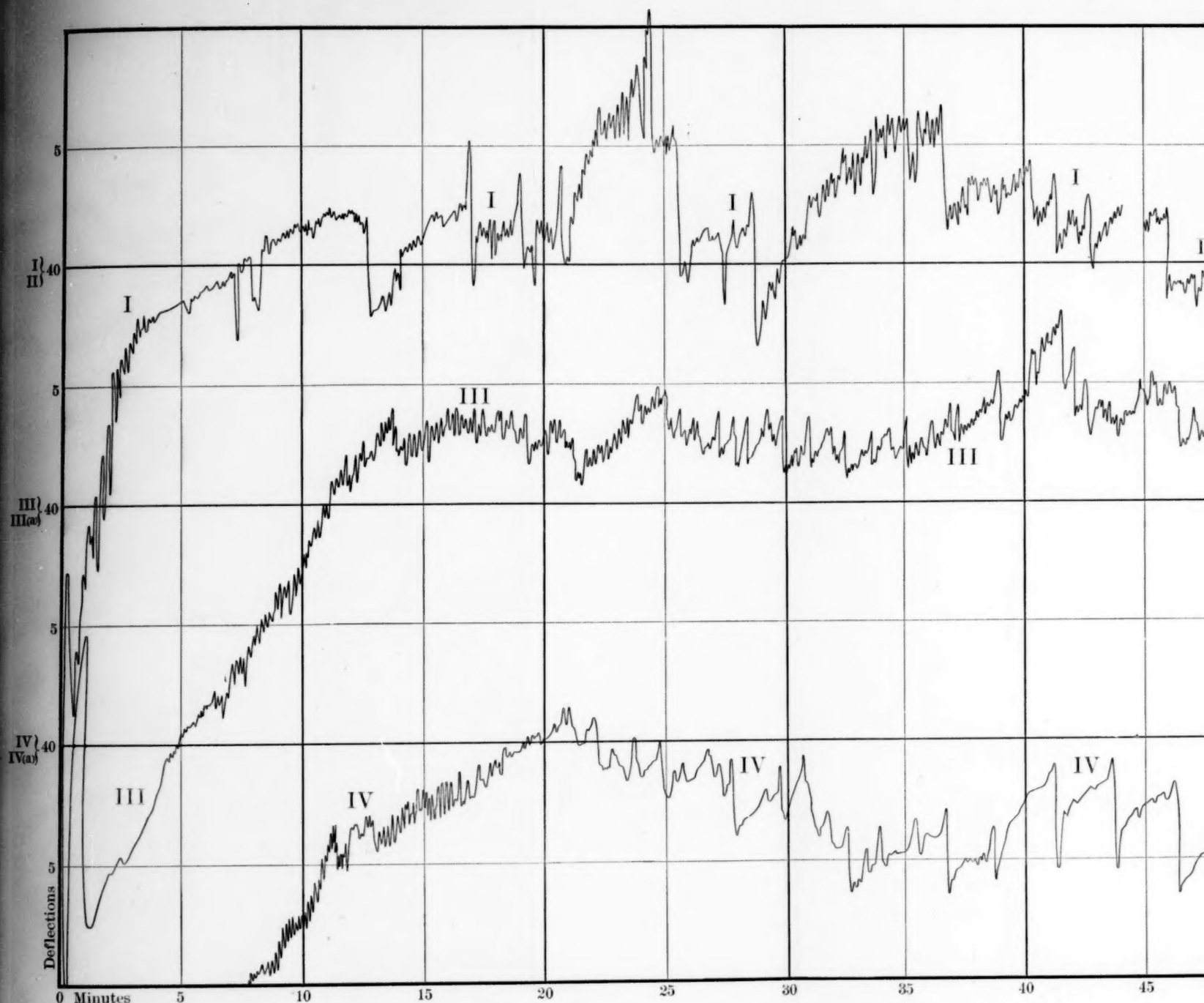
The gravitation constant and the mean density of the earth are functions of the masses and their distances apart. The measurement of these vertical and horizontal distances forms an important part of the work. The method of combining the observations to eliminate the effect of progressive changes deserves special notice. The scale was first read in the initial position, the riders were then moved, and the scale read again; following this, the scale was again read with the riders in the initial position. Then the masses were moved round and the scale read, and finally the reading was made with the masses in the initial position. This program was repeated as long as desirable. Three successive results being taken together, give a value free from progressive changes, and the ratio of the deflection by the mass to that by the rider is based on seven successive centers of swing. The final result obtained is 6.6984×10^{-8} for the gravitation constant, and 5.4934 for the mean density of the earth. Two sets were made, and these treated separately gave for the last quantity the value 5.52 and 5.46. It appears doubtful whether in the final results more than two decimal places are trustworthy. It is understood that the second set with the attracting masses in the opposite position will, to a great extent, eliminate the want of symmetry in the moving parts, but the discrepancies in the results after the interchange would seem to indicate that the final result is given to an accuracy not warranted by the observations. The quantity sought, Δ, is deduced from a formula depending on the distances between the masses and other quantities, such as the earth's compression, centrifugal force, etc. In this formula the difference between the ratios of the mass deflection to rider deflection for the two positions enters as a factor in the denominator. If we examine the values of this factor for the same set, we see differences greater than $\frac{1}{1000}$ part of the whole for the value of A, which really influence the value of $(A - a)$ to nearly the same extent, since a is small. Besides, a has uncertainties of its own, not here considered. If we differentiate the expression for Δ, it appears that the change in the factor $(A - a)$ gives a

change in Δ seven times as great, and that the uncertainties in the measured quantities will make even the second decimal place uncertain. So that 5.49 would seem to be sufficient to characterize the result. It is doubtful whether, with all the various methods hitherto employed, we can assert that the mean density of the earth is known with an error less than one per cent of its value.

The bibliography at the beginning of the work is valuable. At the end appears a catalogue of about seventy pages of scientific and other books inserted by the publishers. We rather deplore this custom of binding extensive catalogues of works, many of them not in any way related to the subject matter of the volume, with scientific books. On the whole, Professor Poynting is to be congratulated on his work. His determination adds another trustworthy value to those already existing, and both the method employed and the care exercised in the observations entitle the result to confidence.

ERASMUS D. PRESTON.

U. OF MICH.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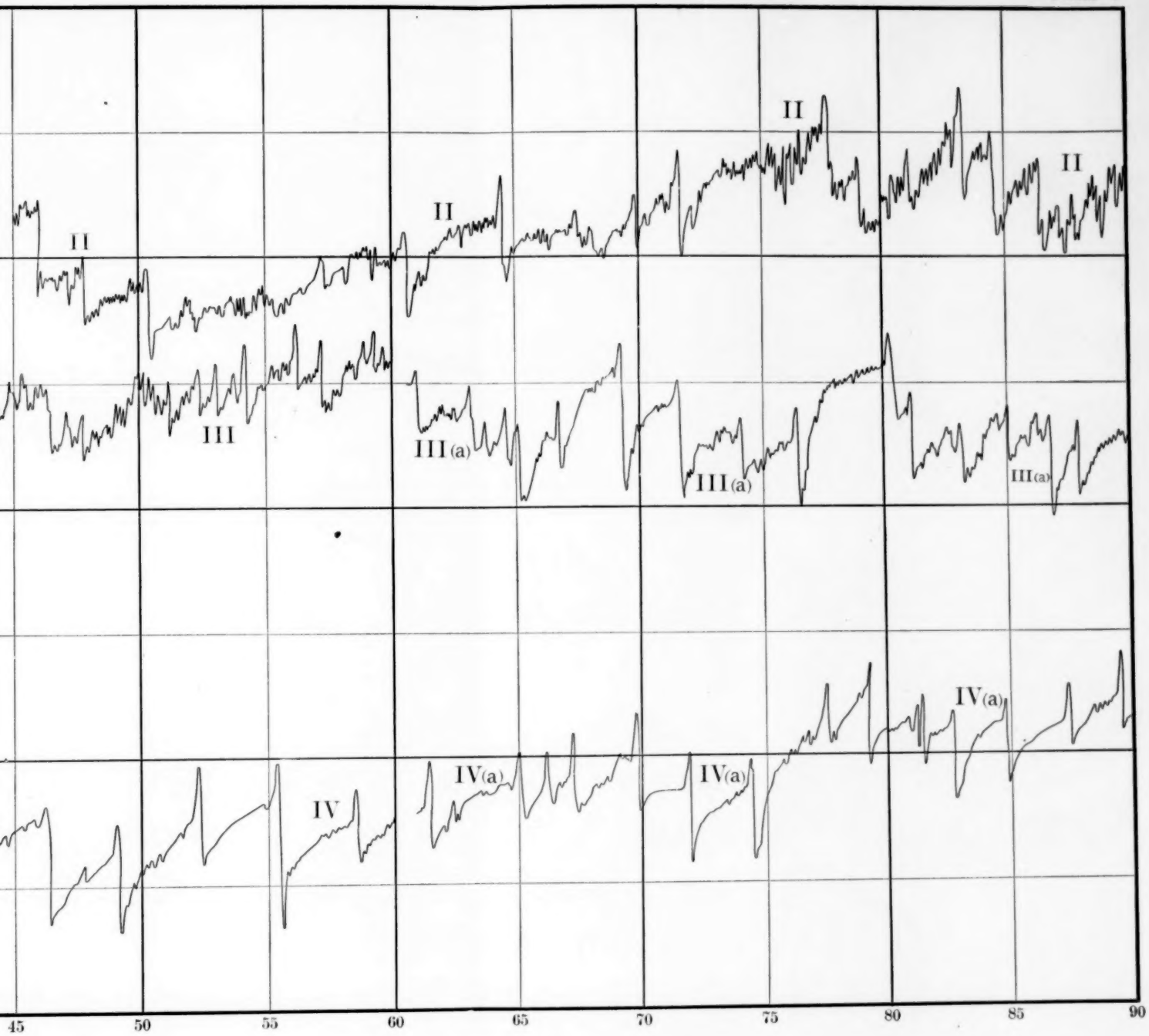
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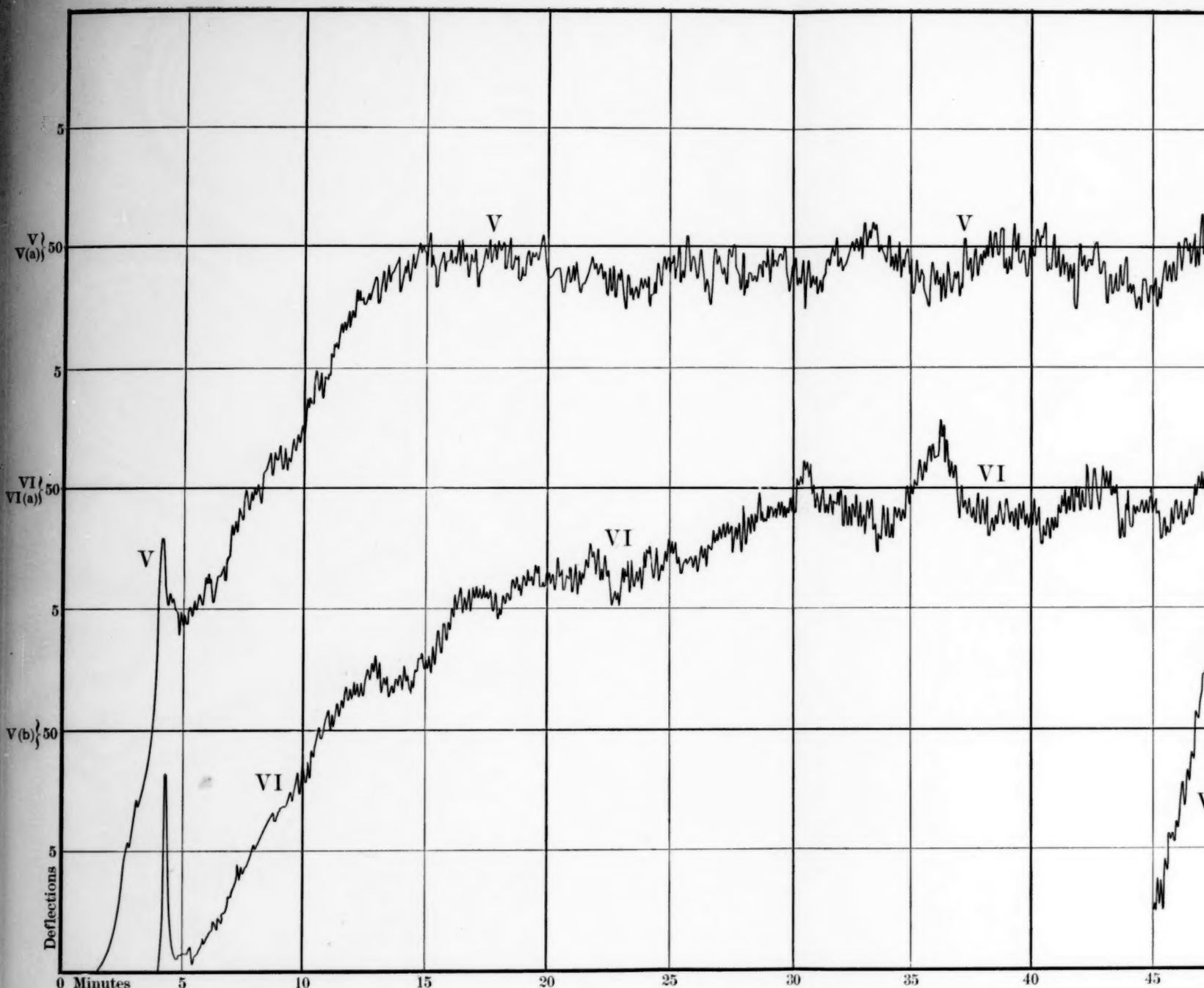
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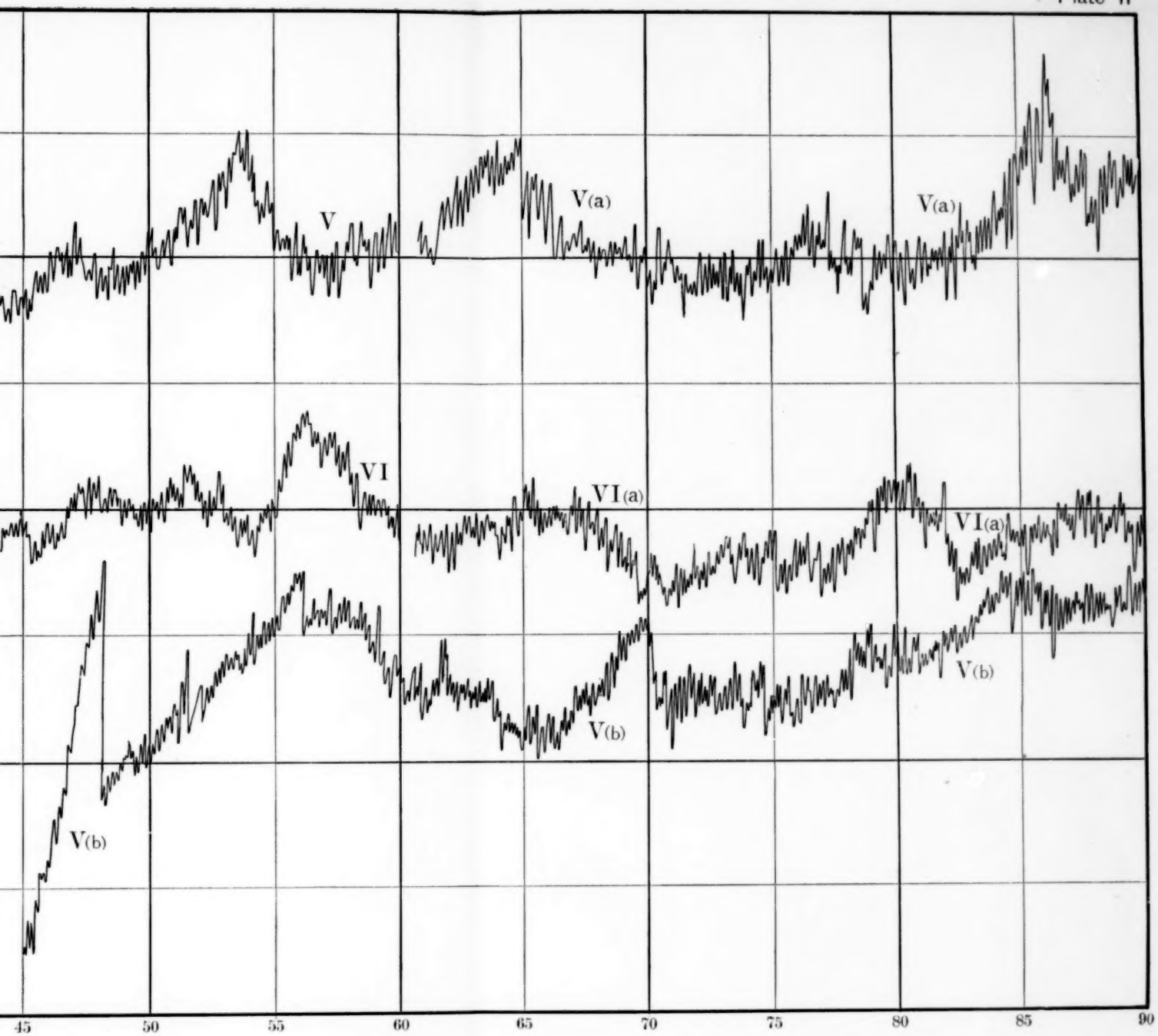
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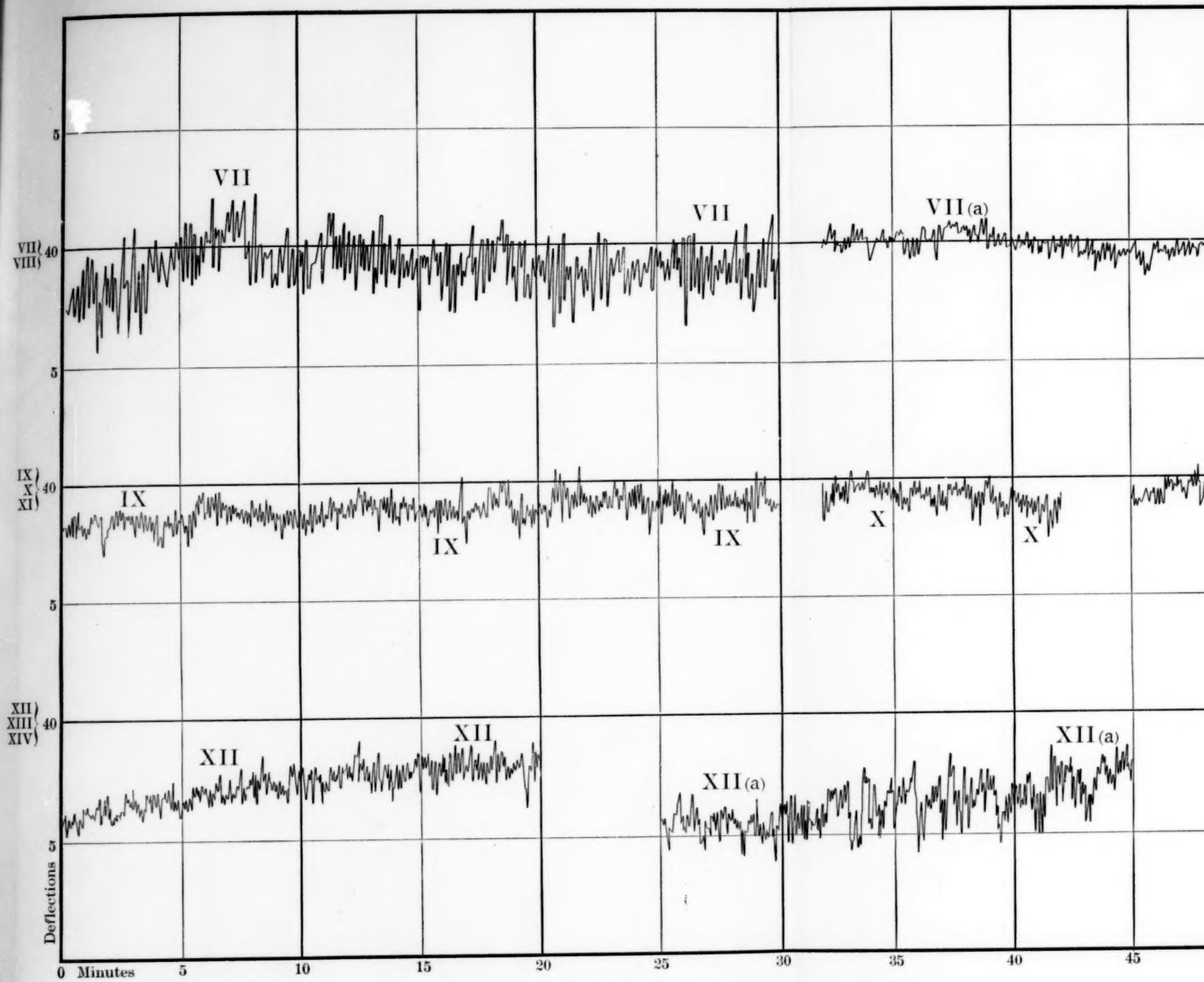
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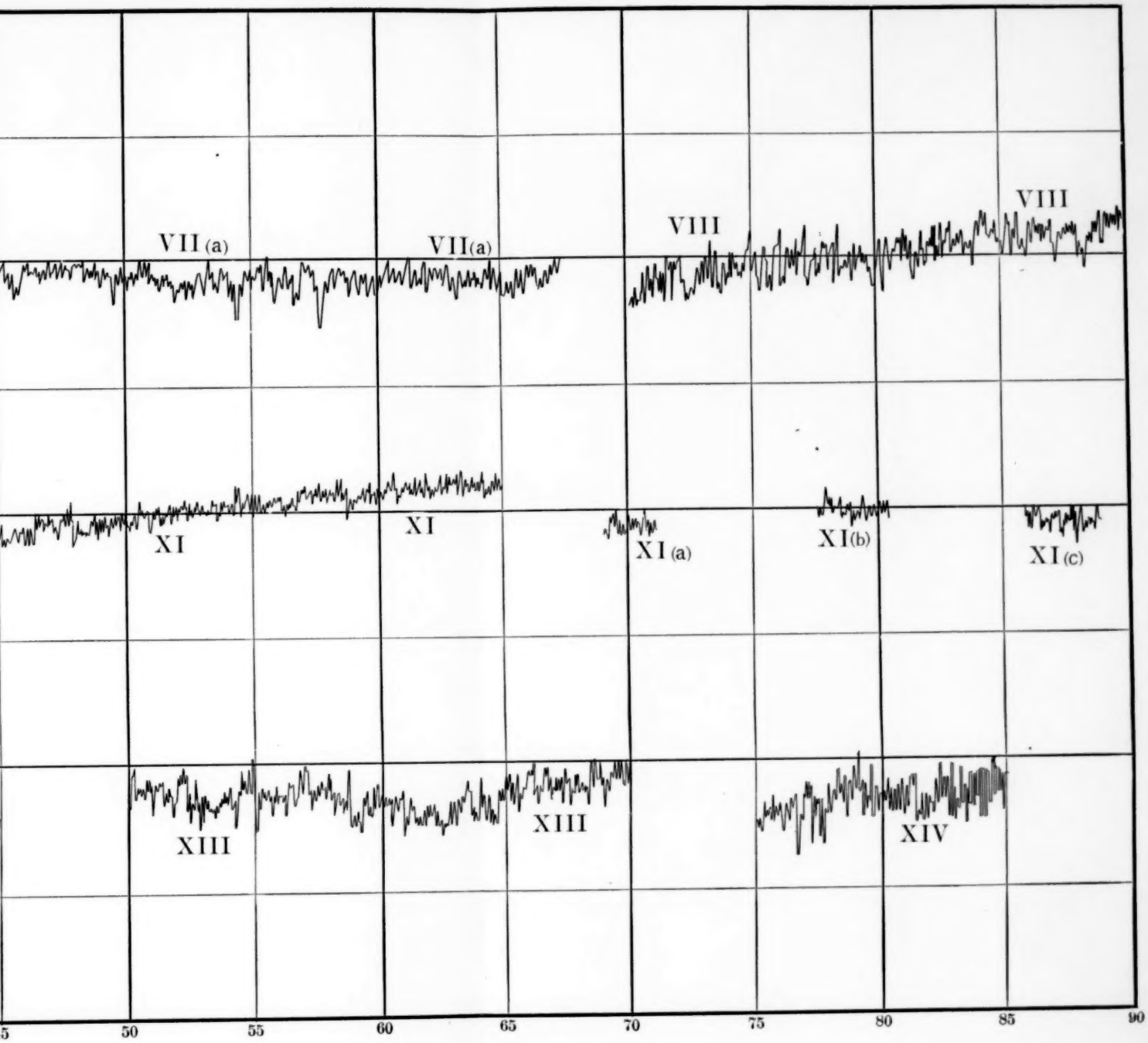
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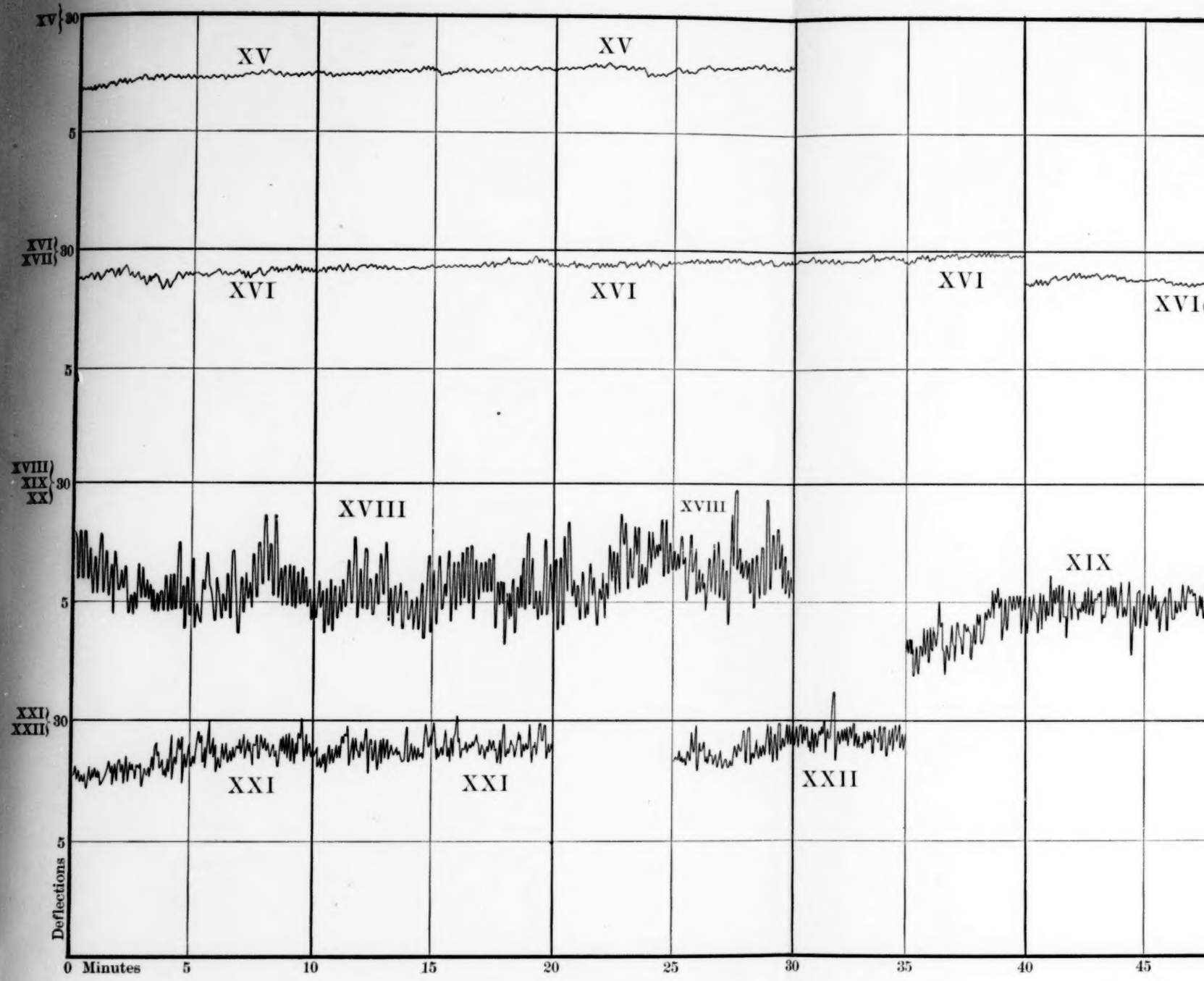
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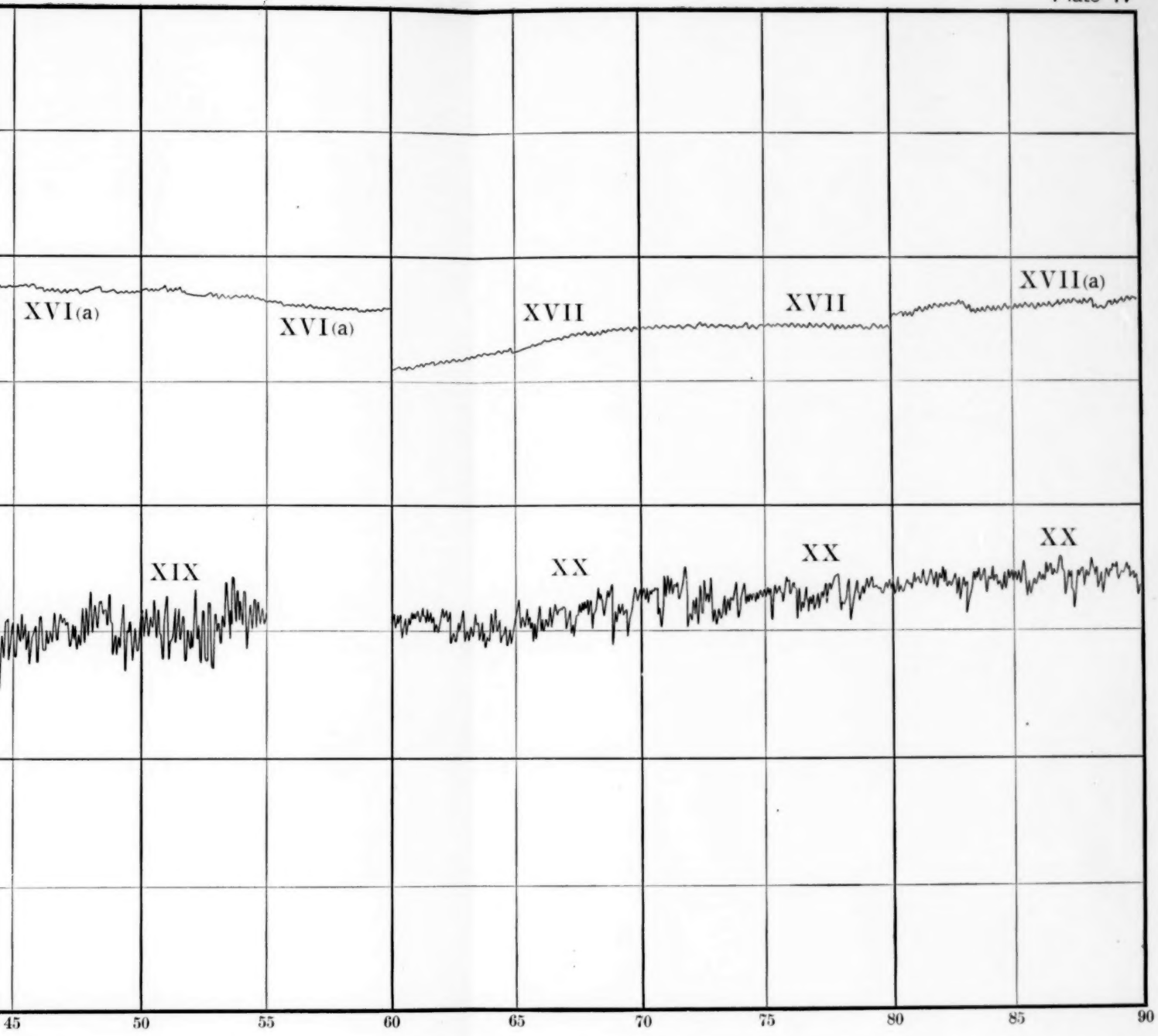
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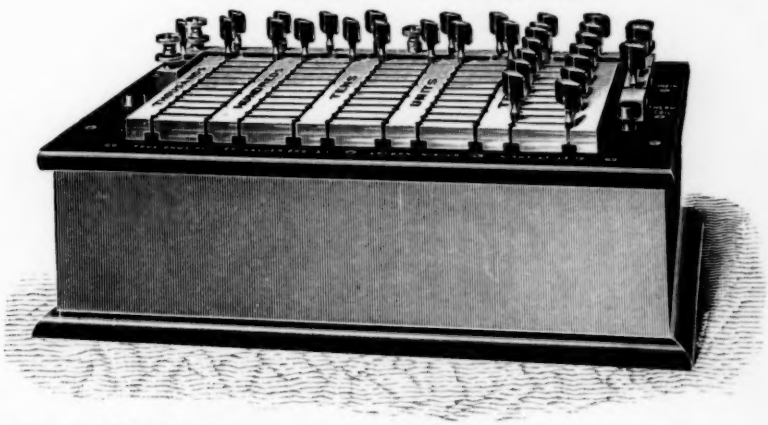

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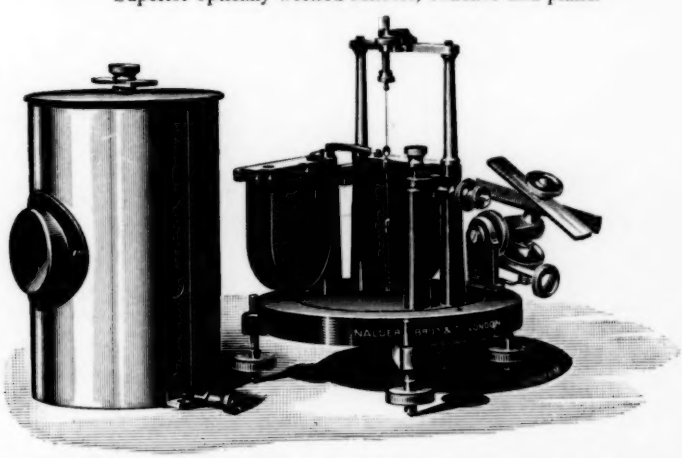

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